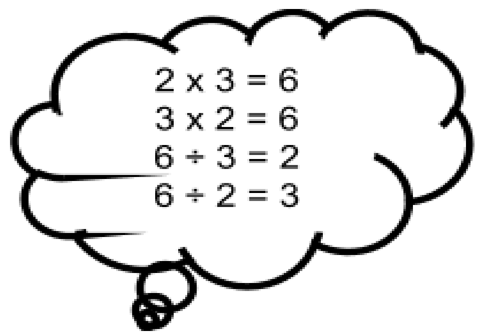
The teaching of multiplication in Chaddlewood Primary School

This calculation policy outlines the progression in mathematical strategies and skills from Foundation to Year 6, and the typical year group children will be in when they are first introduced to particular concepts. It is expected that the majority of children will not draw from objectives in year groups above and below their own. Children will move towards mastery of each of the areas within their year group to ensure that they develop into confident, efficient and accurate mathematicians.

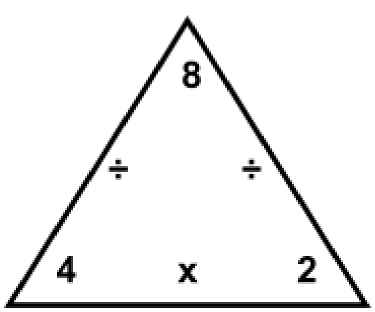
It is essential that, in all year groups, multiplication is:
- taught alongside its inverse division, as these important links will assist children in mastering the operation.
- involved in situations with rich problem solving activities and word problems.
- approached in a cross curricular manner wherever possible.



Children will be given many different types of problems, often which will look very different to what they are used to. This is true for all of the mathematical strategies throughout the calculation policy. For example, in calculating problems involving a missing number (for example 4 x 3 = □), children will also consider:

□ x 3 = 12   4 x □ = 12   12 = 3 x □   12 = □ x 4

To help to develop the links between multiplication and division the children will also use ‘number trios’. Number trios demonstrate to the children that when they choose a ‘trio’, they can make four number sentences with them, by covering up particular numbers. These will be used even further by considering what would happen if we multiplied or divided each of the numbers by 10 or 100.



2 x 4 = 8
4 x 2 = 8
8 ÷ 4 = 2
8 ÷ 2 = 4

Through this calculation policy, which should be read alongside our other mathematical policies, we aim for every child in our school to become;

- fluent in the strategies covered, including the rapid recall and application of key knowledge (for example times tables)

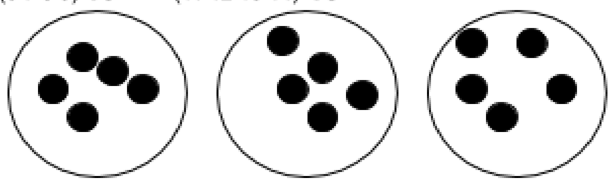
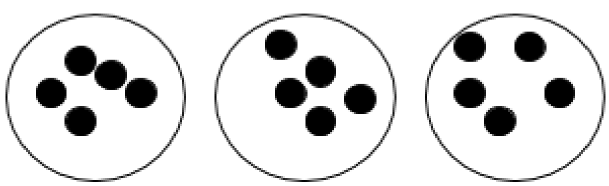
- confident and skilful at reasoning mathematically (including specialising and, eventually, generalising their conceptual understanding).

- efficient at solving problems in a sophisticated manner (for example by breaking down complex problems into simpler steps).

This calculation policy has been written to ensure a seamless progression of skills and strategies. Secondary schools in the local area have been consulted on the content contained within it, and therefore these establishments will be prepared to develop upon the knowledge and understanding that the children have when they leave our school.

Lastly, calculators will only be used when the children are secure with the appropriate strategies outlined within their year group. Calculators are a key tool in saving time with more complex problems, but they will not be used as a replacement for a thorough understanding of the underlying processes involved in calculations.

Key Vocabulary for Place Value	Key Vocabulary for Representations
Ones Tens Hundreds Thousands Ten Thousand Hundred Thousand Millions Tenths Hundredths Column Row Place holder Digit/ Integer	Bar Model Part/Part whole Dienes Base 10 Counters Cubes Bead bar/string

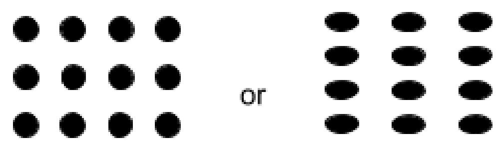
Strategy	***Rationale***
Adding groups by using equipment Multiplication is introduced through problems involving 'lots of' objects. ***Example*** 3 lots of 5 = **5** (6 7 8 9) **10** (11 12 13 14) **15** 	Children will physically make sets or groups, and then add them together by counting up from 1 (until all of the objects have been used).
Adding groups The multiplication sign (x) will be introduced as a short way of saying 'lots of'. ***Example*** 3 x 5 = There are 5 buns on a plate. How many buns are on 3 plates?  **5 … 10 … 15**	Dots are often drawn in groups. This shows how many 'lots of' there are. These groups are then added together

Representing multiplication with arrays

The product of two numbers will be shown using an array (with support from the class teacher). In doing so the children will identify the commutatively of multiplication.

Example

4 x 3 = ?
A chew costs 4p. How much do 3 chews cost?



4 x 3 = 12
and 3 x 4 = 12

Drawing sets gives the children an image of the answer. It also helps them to see that the numbers are reversible (commutative)

This stage begins to showcase how jottings are essential in mathematical problems.

Key vocabulary
Count in twos, threes, fives

Count in tens (forwards from/backwards from)

Once, twice, three times, five times

Lots of, groups of

How many times?

Repeated addition

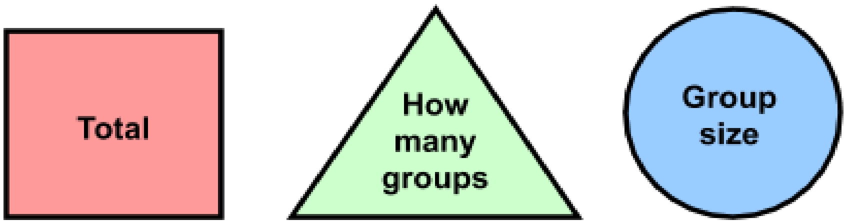
Array, row, column

Mathematical symbols

The children will be introduced to the appropriate mathematical symbols at this stage. They should be able to write mathematical statements involving the multiplication (x) and equals (=) sign.

Elements of multiplication

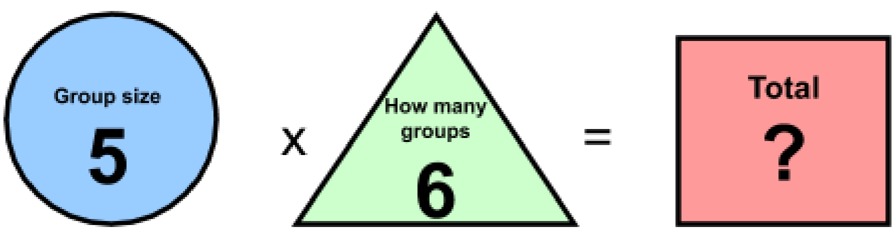
At this stage the children will be introduced to three different elements:



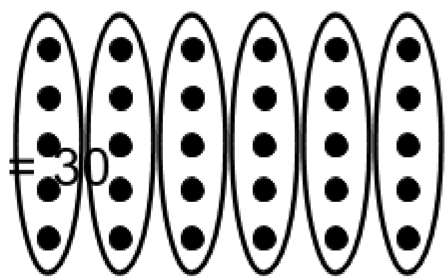
They will understand that these elements can be linked to any given multiplication problem, in order to establish what they need to find out.

Example (first approach)

5 x 6 = ?



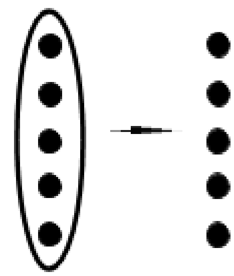
Therefore the jotting will look like the following:



The children will develop their understanding of how the numbers representing the 'group size' and 'how many groups' can be interchanged.

In Year 2, these are linked to 'number trios' (please see above for more information regarding these)

As the children become more confident with their jottings, the group 'ring' may be omitted.



Children will establish the 'total' by initially counting on in ones. They will, however, quickly develop their ability to count in different amounts (e.g. in the particular group size, or in 2s/10s if this is more appropriate).

Children will understand that multiplication is commutative.

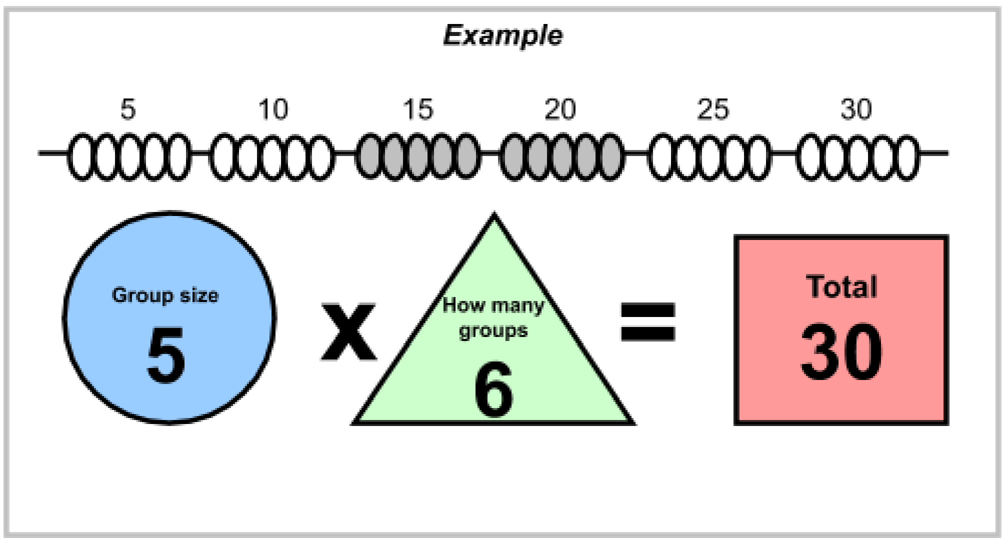
Key vocabulary

Double, near double

Half, halve

Odd, even

Multiply, multiply by

Hundred squares Children will notice patterns in hundred squares, as they begin to count up in different amounts. They will begin to identify which number will come next in a sequence.	This is an introduction to more formal 'times tables' which are particularly emphasised in Years 2, 3 and 4..
Bead strings In preparation for the next step, the children will learn how to count on in 2s, 5s and 10s using a bead string. They will then identify the appropriate elements of the multiplication calculation (as outlined in the 'Elements of multiplication' section above)  	

Number lines

The children will use a number line to help them to 'count up' in different amounts, much like in the repeated addition evidenced in the number arrays. This will initially be using marked number lines.

Example

5 x 4 = ?

So 5 x 4 = 20

The children will be encouraged to jot down / highlight important numbers along the way, especially the number of 'jumps' they have done.

Times table knowledge

Children are expected to know their 2x, 5x and 10x table. By 'knowing' a times table we mean that each child is able to recall instantly any multiplication <u>or division</u> question posed to them for their particular times table. It must not matter to each child which order the calculation is posed to them in (for example 2x6 or 6x2). They must know each fact up to x12, and their division facts (such as 16÷2 or 55÷5)

They are awarded a 'bronze' badge to celebrate their mastery of all three of these times tables.

Times table knowledge

Children are expected to know their 3x, 4x, 8x and 11x table. By 'knowing' a times table we mean that each child is able to recall instantly any multiplication <u>or division</u> question posed to them for their particular times table. It must not matter to each child which order the calculation is posed to them in (for example 3x6 or 6x3). They must know each fact up to x12, and their division facts (such as 16÷4 or 55÷11)

Doubling should be seen as of key importance when initially calculating the 8x times table.

They are awarded a 'silver' badge to celebrate their mastery of the 3x and 4x times tables.

Vertical method

3

This vertical method shows how '6 lots of 24' is the same as:
(6 lots of 4) + (6 lots of 20)

Example

```
  24
 x 6
  24   (6 x4)
 120   (6 x 20)
 144
```

Short multiplication

Short multiplication will then be introduced. This involves multiplying any whole number by a single digit number.

Year 3 children will focus on multiplying a two-digit number by a one-digit number.
Year 4 children will multiply two and three-digit numbers by a one-digit number.

Example

46 x 9 = ?

First, the 'unit' digit of 46 is multiplied by 9. As this results in a two-digit number, the 'tens' are then placed in the 'tens' column.

```
 4 6
 x 9
   4
 5
```

Next, the 'tens' digit of 46 is multiplied by 9 (making 36 tens). Since there is also 5 tens in this column from the first step, these are added to the total. We now have 41 tens (410), which can also be written as 4 hundreds, and 1 ten.

```
   4 6
   x 9
   1 4
 4 5
```

As there are no 'hundreds' digit in 46 to multiply by 9, we record a 4 in the hundreds column (this was from the last step).

```
   4 6
   x 9
 4 1 4
 4 5
```

If asked at any stage the children will be able to identify what each digit in a calculation represents (for example whether it represents so many 'thousands', 'hundreds', 'tens' or 'units').

Year 4 children may need to revisit the 'Vertical Method' stage above and apply it to a three-digit number to emphasise the place value of each recorded digit.

Note that the 'tens' and 'hundreds' digits that are 'carried over' must be written underneath the last horizontal line.

Scaling and Distributive law

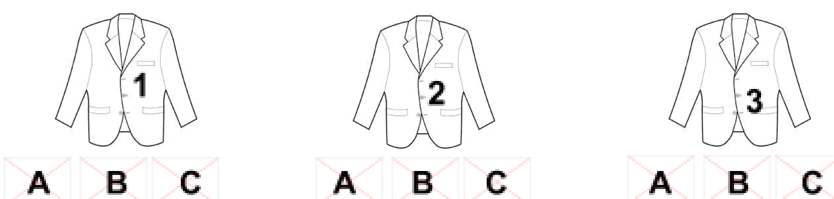
Children will solve different problems involving scaling and correspondence. In doing so, they will develop their ability to apply multiplication much more flexibly.

Example

Children will see that the longer ribbon is four times the length of the shorter, rather than 15cm longer.

Example

A boy has 3 coats and 3 hats.
How many different outfits could he wear?

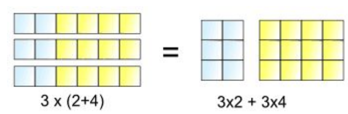


So there are 9 different outfits (1A, 1B, 1C, 2A, 2B, 2C, 3A, 3B, 3C)

3 x 3 = 9

In Year 4, children are also introduced to the Distributive Law. The Distributive Law shows that multiplying a number by a group of numbers added together is the same as doing each multiplication separately

For example,
$3 \times (2 + 4) = 3\times2 + 3\times4$
Therefore the "3" can be "distributed" across the "2+4" into 3 times 2 and 3 times 4.



Key vocabulary
Product

Multiples of four, eight, fifty and one hundred

Scale up

Associative law

Example

2 x 3 x 4 = ?

This can be calculated by carrying out either of the following;
(2 x 3) x 4, so 6 x 4 = 24
or… 2 x (3 x 4), so 2 x 12 = 24

Times table knowledge

Children are expected to know their all of their times table by the end of this stage. By ‘knowing’ a times table we mean that each child is able to recall instantly any multiplication <u>or division</u> question posed to them for their particular times table. It must not matter to each child which order the calculation is posed to them in (for example 7x6 or 6x7). . They must know each fact up to x12, and their division facts (such as 56÷8 or 54÷6)

The children will recognise what happens to any number when it is multiplied by 0, and by 1. They will begin to multiply three digit numbers together, using all of their times tables in doing so to support them.

They are awarded a ‘gold’ badge to celebrate their mastery of all the 6x and 7x times tables, and a ‘times table champion’ badge to celebrate mastery of every other times table (they will therefore at this stage be proficient in their recollection of every times table up to 12x12).

Key vocabulary
Multiplication facts (up to 12x12)

Division facts (Inverse)

Derive

Multiplication by 10, 100, and 1000

The children will learn what happens to a number when it is multiplied by 10, 100 and 1000. This will consolidate their place value, and prepare them for when they need to multply by a decimal number.

Long multiplication

This strategy builds on the short multiplication method above, but this time introduces larger numbers being multiplied together (where both numbers are greater than 9) Children will use up to 4 digit numbers.

Example

46 x 19 = ?

	4 6	
	x 1 9	
Step 1: Multiply tens x units	3 6 0	(40x9)
Step 2: Multiply units x units	5 4	(6x9)
Step 3: Multiply tens x tens	4 0 0	(40x10)
Step 4: Multiply units x tens	6 0	(6x10)
Step 5: Add the answers together	8 7 4	
	1	

Once confident with this method, children will be shown how to combine steps 1 and 2, and steps 3 and 4 (see below).

Please note the specific order in which the digits are multiplied. The digits with the largest value should be multiplied first so that the children do not 'forget' to include the '0s' in their recorded lines (and hence be incorrect by a multiple of ten). For example, tens and always multiplied before units are.

Key vocabulary

Formal written method

Shorter multiplication methods

Example

46 x 19 = ?

	4 6	
	X 1 9	
Step 1: Multiply top amount x tens	4 6 0	(46x10)
Step 2: Multiply top amount x units	4 1 4	(46x9)
Step 3: Add the answers together	8 7 4	

This step involves reducing the above method into two stages.

Year 6 children will move towards multiplying a four-digit number by a two-digit number. Their method will therefore contain 4 discrete steps (‘x thousands’, ‘x hundreds’, ‘x tens’, ‘x units’ and ‘adding above numbers together’).

Please see the note above regarding the specific order of steps for multiplying the different digits (in this instance, the tens are multiplied before the units).

Brackets

Children will develop an understanding of how to approach problems which involve the use of brackets, including the mathematical rules underpinning extended number sentences (for example that they should always solve the mathematical calculation within the brackets first, and be able to read problems where mathematical symbols have been omitted).

Children will be introduced to the term 'BODMAS' to represent the order that operations need to be carried out.

Brackets
Order (for example, 'powers' such as 3^2)
Division
Multiplication
Addition
Subtraction

Notice that, in the first example below, the solution is different depending upon where the brackets are placed.

Example

(7 + 2) x 3 = (9) x 3 = 27

7 + (2 x 3) = 7 + (6) = 13

Example

3(7+2) = 3(9) = 27

In these examples brackets are used to increase the complexity of the calculation.

Key vocabulary
Order of operations

Multi-step problems and decisions about:
(i) which operation to use
(ii) the degree of accuracy in each calculation

At this stage the children will be proficient at being able to identify when they are required to multiply a set of quantities. They will be able to recognise elements to multiply, even in problems which involve multiple 'steps'.

The children will build upon the vocabulary work that they have previously experienced. Mastery of this concept will also be developed through the use of visually representing particular problems (through the use of jottings).

It the children do not need to multiply in a calculation, they will still be able to identify which other operation to use.

Formal written multiplication method, with decimals

Example

52 x 4.3 = ?

	5 2	
	x 4 .3	
Step 1: Multiply top amount x units	2 0 8 .0	(52x4)
Step 2: Multiply top amount x decimal	1 5 .6	(52x0.3)
Step 3: Add the answers together	2 2 3 .6	

This step will then be expanded to include numbers with a great number of digits, including numbers with up to two decimal places.

Please refer to the above steps in order to ensure that the children multiply the digits in a specific manner (helping to prevent potential errors).

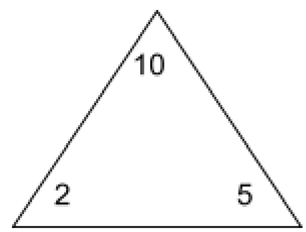
Multiplication of fractions and mixed numbers

Example

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$2\frac{1}{2} \times \frac{1}{4} = \frac{5}{8}$ *because* $\frac{5}{2} \times \frac{1}{4} = \frac{5}{8}$

When multiplying an integer with a fraction, number trios will be used to identify the relationship between all numbers.



1/5 x 1/2 = 1/10
1/2 x 1/5 = 1/10
(and 10 x 1/2 = 5)
(and 10 x 1/5 = 2)
1/10 ÷ 1/2 = 1/5
1/10 ÷ 1/5 = 1/2

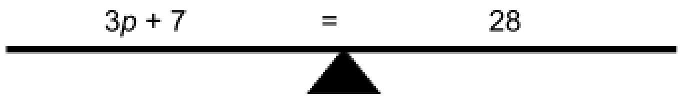
Algebra

The children will learn that algebra involves the use of simplified number sentences, where both sides of the equals sign needs to 'balance'.

Example

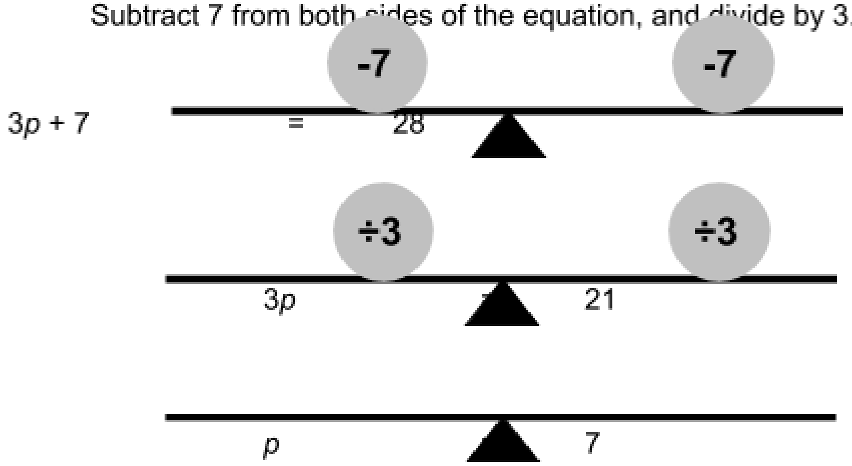
$3p + 7 = 28$

In this case *p* represents a missing number, but any letter could be used. Imagine the number sentence, balanced on a see-saw.



To keep the see-saw balanced whatever happens to one side must happen to the other. As we need to find the value of *p* our goal is to isolate this letter, with all of the numbers are on the other side of the equation.

Subtract 7 from both sides of the equation, and divide by 3.



Algebra may look confusing, but it is simply way of representing a missing number with a letter.

The children have tackled problems similar to this much earlier in their scho lives, for example in 'missing number' sentences. The missing number boxes are now ju replaced with a mathematical symbol.

□ + 6 = 15
f + 6 = 15

Extended algebra

Example

$4f + 7 = f + 16$

$4f + 7 – 7 = f + 16 – 7$ (subtract 7 from both sides)

$4f = f + 9$

$4f – f = f + 9 – f$ (subtract *f* from both sides

$3f = 9$

$f = 3$ (divide both sides by 3)

This example is more involved. If you imagine th left and right hand sides o this problem being balanced, like on a see-saw, then you can kee the balance by doing the same to both sides.