The teaching of subtraction in Chaddlewood Primary School

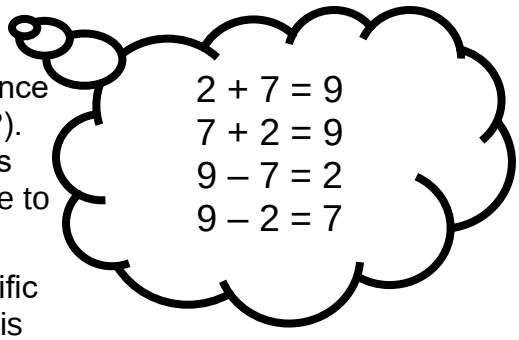
This calculation policy outlines the progression in mathematical strategies and skills from Foundation to Year 6, and the typical year group children will be in when they are first introduced to particular concepts. It is expected that the majority of children will not draw from objectives in year groups above and below their own. Children will move towards mastery of each of the areas within their year group to ensure that they develop into confident, efficient and accurate mathematicians.

It is essential that, in all year groups, subtraction is:
- taught alongside its inverse addition, as these important links will assist children in mastering the operation.
- involved in situations with rich problem solving activities and word problems.
- approached in a cross curricular manner wherever possible.

There are two key elements to subtraction:

1) 'Finding the difference' questions, which involve finding the difference between two quantities (for example 'how much bigger is 7 than 3?). These problems normally comprise of words ending in 'er', such as bigger, smaller, taller and shorter. Often it is more common for people to want to count up from the smallest to the largest quantity

2) 'Finding what's left' questions, which involve 'taking away' a specific quantity in order to establish how much is left. In these problems it is more common for people to count backwards in finding a solution.

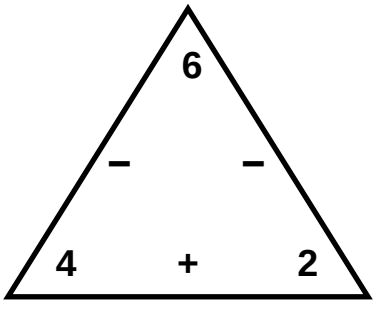


Throughout our school both of these key elements are addressed side-by-side, with children being encouraged to choose an appropriate method for working out problems (for example, should they jump forwards or backwards on a number line?)

Children will also be given many different types of problems, often which will look very different to what they are used to. This is true for all of the mathematical strategies throughout the calculation policy. For example, in calculating problems involving a missing number (for example 10 – 3 = ☐), children will also consider:

☐ – 3 = 7 10 – ☐ = 7 7 = 10 – ☐ 7 = ☐ – 3.

To help to develop the links between addition and subtraction the children will also use 'number trios'. Number trios demonstrate to the children that when they choose a 'trio', they can make four number sentences with them, by covering up particular numbers. These will be used even further by considering what would happen if we multiplied or divided each of the numbers by 10 or 100.



2 + 4 = 6
4 + 2 = 6
6 – 2 = 4
6 – 4 = 2

Through this calculation policy, which should be read alongside our other mathematical policies, we aim for every child in our school to become;

- fluent in the strategies covered, including the rapid recall and application of key knowledge (for example times tables)

- confident and skilful at reasoning mathematically (including specialising and, eventually, generalising their conceptual understanding).

- efficient at solving problems in a sophisticated manner (for example by breaking down complex problems into simpler steps).

This calculation policy has been written to ensure a seamless progression of skills and strategies. Secondary schools in the local area have been consulted on the content contained within it, and therefore these establishments will be prepared to develop upon the knowledge and understanding that the children have when they leave our school after Year 6.

This document should also be read in conjunction with our 'models and images' policy for the appropriate operation. The policy draws upon the schools' 'Concrete, Pictorial, Abstract' approach, which emphasises the importance of mastery and the use of different representations, including through the use of conceptual variation.

Lastly, calculators will only be used when the children are confident and secure in their understanding of a concept outlined below. Calculators are a useful tool in assisting thought processes beyond timely calculation methods. They will not, however, be used as a replacement for conceptual and intrinsic understanding of each strategy covered in this policy.

Key Vocabulary for Place Value	Key Vocabulary for Representations
Ones Tens Hundreds Thousands Ten Thousand Hundred Thousand Millions Tenths Hundredths Column Row Place holder Digit/ Integer	Bar Model Part/Part whole Dienes Base 10 Counters Cubes Bead bar/string Rekenrek

Strategy	Rationale
Using songs and number rhymes ***Example*** 'Five little speckled frogs' 'Five little men in a flying saucer' '10 Green Bottles' Subtraction vocabulary will be introduced. This includes 'take away', 'less than' and 'subtract'. Children will be initially taught that, when subtracting, the answer is smaller than the starting number.	Children will use their counting skills to find 'one less' than a quantity, using their fingers to help them to count (from 10). Children will also use objects, pictures, stories and songs to help develop their understanding. They will count and point using objects, and subtract them by physically moving each item. Whenever possible we use real life experiences to develop the children's understanding of subtraction.
Subtraction using pictures ***Example*** I have 5 balloons. 2 burst. How many do I have left?  The formal method of recording will be introduced. ***Example*** 5 – 2 = 3 The children will continue to develop their understanding of the vocabulary associated with subtraction.	Using a picture helps children to visualise the problem (for example they could cross out each item as it is 'taken away'). The children will respond to questions like 'How many are left?'

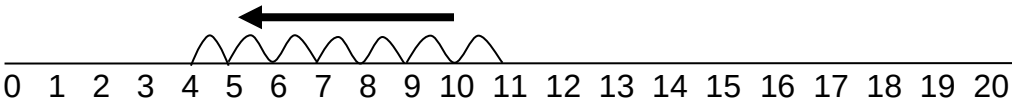
Subtraction using a number line

Children will develop their ability to subtract (by 'taking away'). This will involve them jumping backwards on a number line. This will prepare them to deal with larger quantities, and it will also become more time efficient.

Example

What is 11 take away 7?

11 – 7 = ☐



Start on 11, and 'jump back' 7 spaces.

11 – 7 = 4

Children start using a number line to subtract only one at a time (not tens).

Key vocabulary

Difference between
How many more to make..?, how many more is…than..?, how much more is..?

Subtract, take away, minus

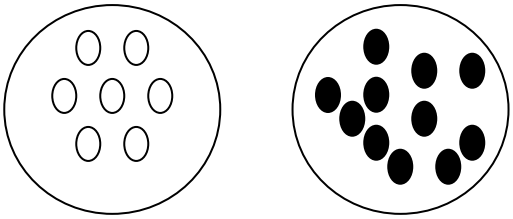
How many fewer is…than..?, how much less is..?

Subtraction by finding the difference

After having experienced subtraction as 'taking away', the children will be introduced to subtraction as 'finding the difference'.

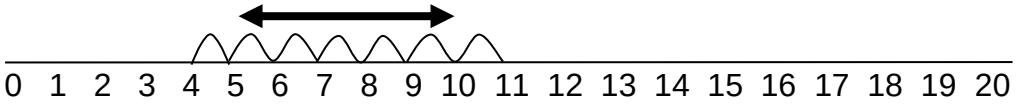
Example

Which plate holds the most: black or white?
How many more? What is the difference?



Example

What is the difference between 11 and 4?



Children will start on either number and 'jump' until they reach the other (this is possible in either direction, but is normally thought of as 'jumping on').

Finding the difference is introduced pictorially. The first example shows how this type of problem is solved by counting the objects to find how many more are needed.

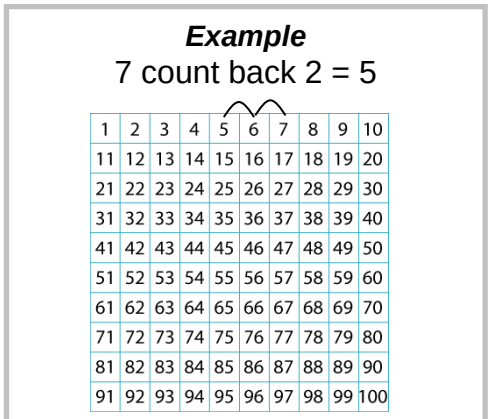
Children can also solve subtraction problems by counting on or counting back using a number line. The method that the children will use will depend upon how they 'see' a problem. If a child is stuck on the problem they will be encouraged to count on from the smallest number (as this is normally the case when, for example, finding the difference between the heights of 2 people).

Introducing a hundred square

Children will use a hundred square to 'jump back' particular amounts.

Example

7 count back 2 = 5



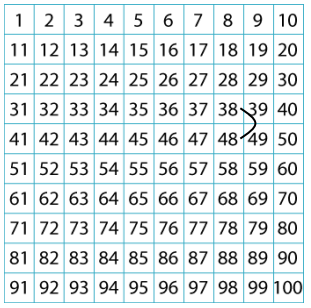
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

7 – 2 = 5

They will then move to a more efficient method of subtracting 10 to a number (jumping vertically rather than horizontally).

Example

48 count back 10 = 38



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

48 – 10 = 38

Children begin to use 100 squares as a tool to aid counting back in small steps (eg. in 1s or 2s)

Once secure they begin to use the 100 square to count back in tens.

Children learn that as they move down a row they subtract 10 each time.

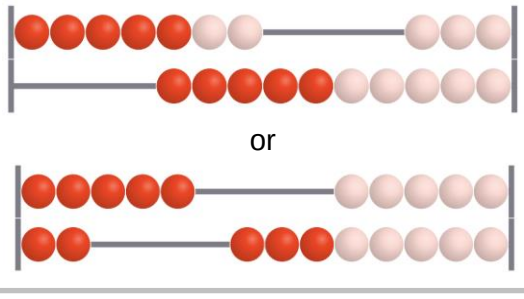
Careful attention is given to possible misconceptions at this stage, especially jumping on their starting number, instead of always moving horizontally with each move.

Using rekenrek

The children will use rekenrek as a key tool to compose and decompose different numbers.

They will be able to represent numbers in varying ways using this frame.

Example
7 = 5 + 2



or

The children will learn how to describe the image to the left using stem sentences such as

'7 is the whole. 5 is a part and 2 is a part'

and

'5 needs 2 to make 7'.

Introducing partitioning (2 digit numbers)

Children will learn that numbers 10 or over (and under 100) are made up of TENS (left hand digit) and ONES (right hand digit).

Partitioning a number involves splitting it up into TENS/ONES to show the value of each digit.

Example
34 comprises of:

3 (tens) 4 (ones)

Initially this will be practically done using 'Deines' (which comprise of sticks representing 'tens', and cubes representing 'ones').

This method is also used when children are introduced to the idea of adding HUNDREDS.

As children become secure they will say the value of each digit without apparatus.

More complex subtraction using a hundred square

Prior to using the hundred squares below the children will need to have a secure understanding of the value of each digit in a number, as determined by its position. This is called ***place value****.*

Using a hundred square

Example

58 – 12 = ☐

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

58 – 12 = 46

Subtracting 12 involves moving up a row, and then to the left 2 places.

Children learn to use hundred squares to subtract 2 digit numbers.

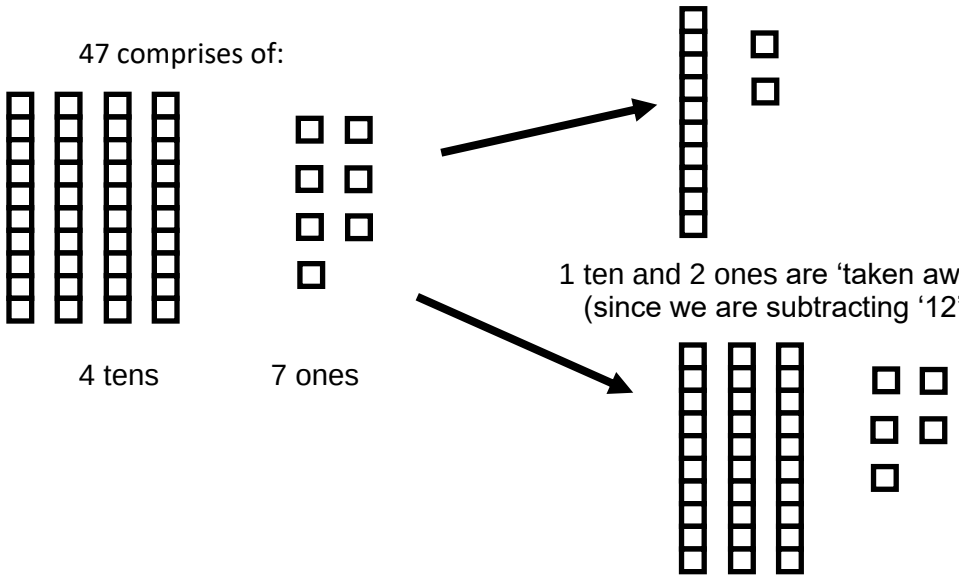
The children will be able to use a hundred square to 'find the difference' and to 'take away' as they gain in confidence with using it.

Subtraction using place value

A quantity can then be subtracted by first partitioning the starting number into 'tens' and 'ones', and then removing a particular number of these (according to the amount to be subtracted).

Example

47 – 12 = ☐



Children will be taught to subtract large numbers using partitioning. This can be done using sticks (representing the 'tens') and single cubes (representing the 'ones')

Children will need lots of practical experience and discussion to understand this process.

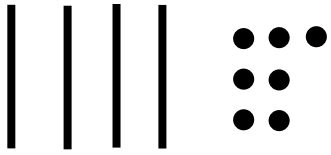
Subtraction using jottings

The efficiency of the above method is then improved even further. The children will be encouraged to represent their workings using a series of jottings showing 'sticks' (to represent tens) and 'dots' (to represent ones).

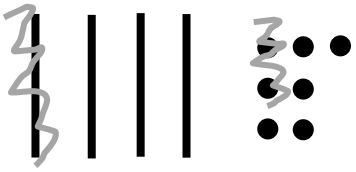
Example

47 – 12 = ☐

"47 is made up of 4 TENS and 7 ONES"



"I need to subtract 12. That's 1 TEN and 2 ONES"



"I have 3 TENS and 5 ONES left, so the answer is 35"

Children initially draw the appropriate number of 'sticks' and 'dots' to represent their starting number. They then either rub out, or scribble out, the amount to be subtracted.

Whilst not essential at this stage, children can also be introduced to drawing a small 'square' to represent the HUNDREDS block (as this is used whilst using Deines). This can then be used to add and subtract three-digit numbers.

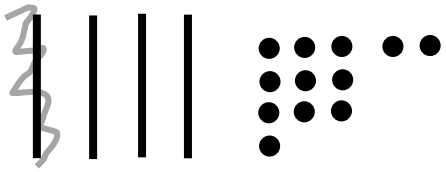
Once mastery of the above has been achieved, children will be introduced to 'exchanging' a TEN for 10 ONES.

Example

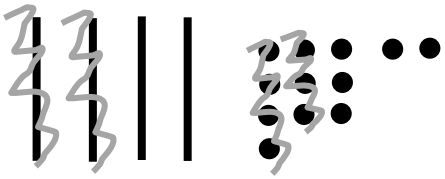
42 – 17 = ☐

"4 is made up of 4 TENS and 2 ONES"

"I only have 2 ONES and need to subtract 7 ONES.
So I will exchange a TEN for 10 ONES"



"I can now subtract 1 TEN and 7 ONES"



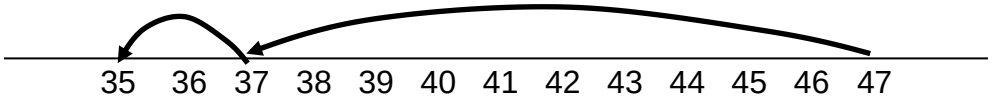
"I have 2 TENS and 5 ONES left, so the answer is 25"

Marked number lines

At this stage the children will learn how to subtract a two-digit number by subtracting the 'tens' and 'ones' within this number using two number line jumps (rather than just jumping back in ones).

Example

47 – 12 =



In this example, one 'ten' is subtracted, followed by two 'ones'.

Using number lines it is easier to take larger jumps (eg. in 10s).

When using a number line, children will display their jumps above the line, and numerals underneath to aid presentation.

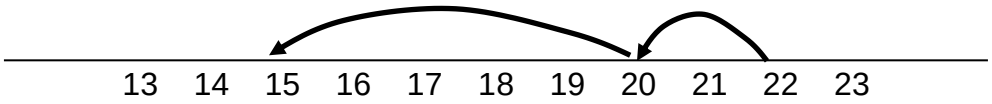
It is important for children here to appreciate that number lines go on infinitely, including into negative numbers.

Noticing shortcuts on number lines

Children will develop their ability to use a number line, including shorter methods to help them when appropriate.

Example

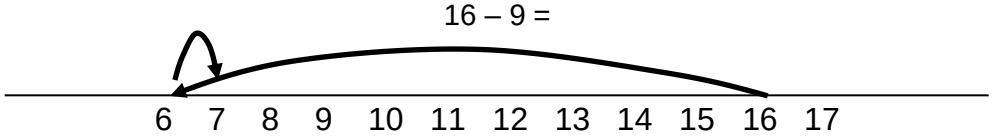
22 – 7 =



In this example, the children will recognise that they are able to jump to the next multiple of ten (subtracting 2), before subtracting the remainder (5).

Example

16 – 9 =



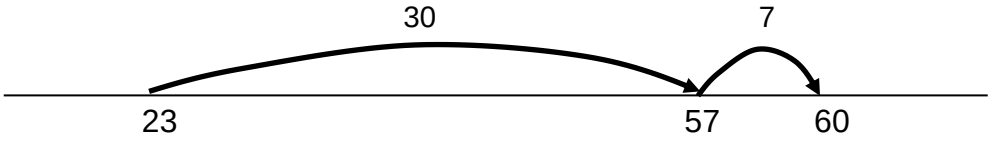
In this example, the children will recognise that '9' is nearly the same as subtracting 10. They will therefore subtract 10, before adjusting their answer to suit the question.

Unmarked number lines

The children will use a blank number line to count on and back in larger amounts, much like the strategies above. They will record significant numbers along the number line, and think carefully about how best to utilise their understanding of multiples of ten to help them when calculating.

Example

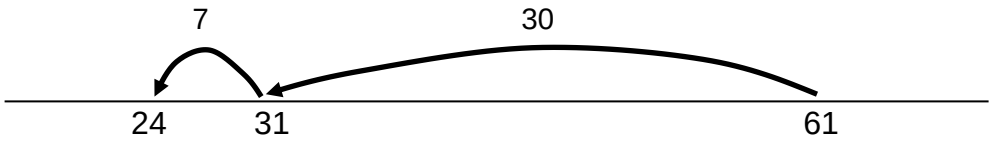
What is the difference between 23 and 60?



so the difference is 37.

Example

What is the difference between 24 and 61?



so the difference is 37.

Notice here that there is no jump to the nearest multiple of ten, but this will be introduced for children who struggle with counting back in tens before moving to the method above.

It is important for the children to display the number of 'jumps' they have made above the number line, so that these can easily be translated into a number sentence afterwards.

Notice that these jumps do not include an addition or subtraction sign to further emphasise how counting forwards or backwards on a number line obtains the same result.

This blank number line strategy will be used to underpin the relationships between addition and subtraction.

The method that the children will use will depend upon how they 'see' a problem. If a child is stuck on the problem they will be encouraged to count on from the smallest number (as this is normally the case when, for example, finding the difference between the heights of 2 people.

Introduction of the formal method for subtraction

The children will subtract a two-digit number from another using a formal written method, which does not involve exchanging.

Example

38 – 13 = ?

38
<u>- 13</u>
<u> 25</u>

so 38 – 13 = 25

Children are introduced to column subtraction without any exchange

This column subtraction follows the more conventional method must adults are familiar with.

Key vocabulary
Column subtraction

Formal written method for subtraction

This process is then repeated for three-digit numbers, with the children experiencing first the calculation without a need to 'exchange'.

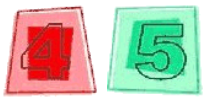
Example

243 – 122 = ?

243
<u>- 122</u>
<u> 121</u>

so 243 – 122 = 121

Formal written method for subtraction (2)



Finally, the children will move on to a formal written method involving ‘exchanging’.

Example

443 – 237 = ?

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{4}\,3 \\ -\,2\,3\,7 \\ \hline 2\,0\,6 \end{array}$$

Example

400 – 199 = ?

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{0}\,{}^{9}\not{0} \\ -\,1\,9\,9 \\ \hline 2\,0\,1 \end{array}$$

At this stage the numbers are no longer restricted.

Exchanging involves a higher digit being exchanged, for example a ‘ten’ being exchanged for ten ‘ones’, or a ‘hundred’ being exchanged for 10 ‘tens’. This then enables a problem to be calculated correctly.

The term ‘borrowing’ must not be used as, for example, the ones are never exchanged back to a ten.

Children will be encouraged to estimate and check their answers by using the reverse calculation.

Key vocabulary

Efficient written method

Predicting solutions and checking their suitability.

Through the use of the above strategy, children will ensure that their solutions are 'reasonable'.

Children will estimate, before calculating, what their solution is likely to be close to. This will involve increasingly precise rounding of numbers, especially in Years 5 and 6, to define the likely boundaries for their solution.

Children will also use the inverse of subtraction (addition) to check when they have doubts about the accuracy of their answer

Example

587
- 315
———

"I know that 600-300 is 300 and so my answer is going to be close to this number,"

moving to…

"I know that 590-320 = 270, and so my answer will be close to this number."

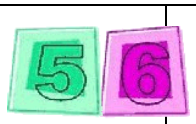
Example

587
- 315
272

"So 272 + 315 = 587"

272
+ 316
587

Multi-step problems and decisions about:
(i) which operation to use
(ii) the degree of accuracy in each calculation



At this stage the children will be proficient at being able to identify when they are required to subtract a set of quantities. They will be able to recognise elements to subtract, even in problems which involve multiple 'steps'.

The children will build upon the vocabulary work that they have previously experienced. Mastery of this concept will also be developed through the use of visually representing particular problems (through the use of jottings).

It the children do not need to subtract in a calculation, they will still be able to identify which other operation to use.

Decimal numbers

Example

34.8 – 26.4 = ?

$$\begin{array}{r} {}^{2}\not{3}\,{}^{1}4.8 \\ -\,26.4 \\ \hline 08.4 \end{array}$$

so 3.8 – 26.4 = 8.4

Example

34.84 – 26.4 = ?

$$\begin{array}{r} 34.84 \\ -\,26.40 \\ \hline 08.44 \end{array}$$

so 3.84 – 26.4 = 8.44

The children will also be given opportunities to subtract numbers with a different number of decimal places (eg. 34.85 – 14.7). The trick here is to put a 0 in any 'spare' space to help the mathematician to put the correct digit in the correct place.

Brackets

Children will develop an understanding of how to approach problems which involve the use of brackets, including the mathematical rules underpinning extended number sentences (for example that they should always solve the mathematical calculation within the brackets first, and be able to read problems where mathematical symbols have been omitted).

Children will be introduced to the term 'BODMAS' to represent the order that operations need to be carried out.

Brackets
Order (for example, 'powers' such as 3^2)
Division
Multiplication
Addition
Subtraction

Notice that, in the first example below, the solution is different depending upon where the brackets are placed.

Example

$(7 - 2) \times 3 = (5) \times 3 = 15$

$7 - (2 \times 3) = 7 - (6) = 1$

Example

$3(7-2) = 3(5) = 15$

In these examples brackets are used to increase the complexity of the calculation.

Key vocabulary
Order of operations

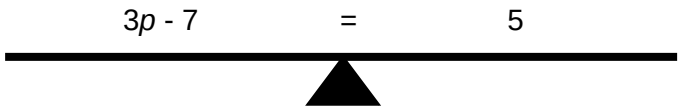
Algebra

The children will learn that algebra involves the use of simplified number sentences, where both sides of the equals sign needs to 'balance'.

Example

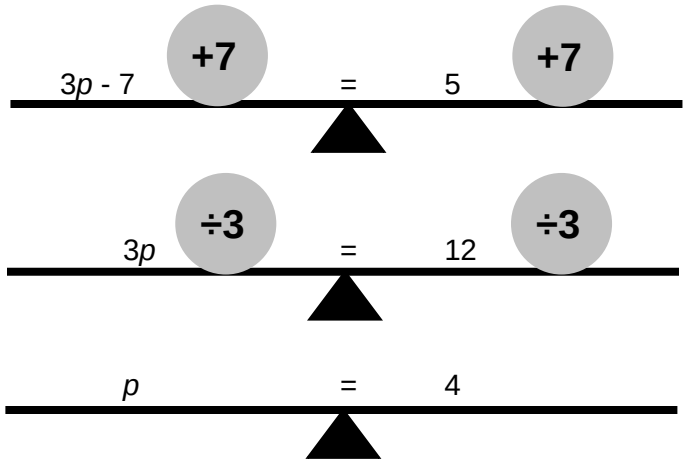
$3p - 7 = 5$

In this case *p* represents a missing number, but any letter could be used. Imagine the number sentence, balanced on a see-saw.



To keep the see-saw balanced whatever happens to one side must happen to the other. As we need to find the value of *p* our goal is to isolate this letter, with all of the numbers are on the other side of the equation.

Subtract 7 from both sides of the equation, and divide by 3.



Algebra may look confusing, but it is simply a way of representing a missing number with a letter.

The children have tackled problems similar to this much earlier in their school lives, for example in 'missing number' sentences. The missing number boxes are now just replaced with a mathematical symbol.

□ - 6 = 1
f - 6 = 1

Extended algebra

Example

$4f - 7 = f + 5$
$4f - 7 + 7 = f + 5 + 7$ (add 7 to both sides)
$4f = f + 12$
$4f - f = f + 12 - f$ (subtract *f* from both sides
$3f = 12$
$f = 4$ (divide both sides by 3)

This example is more involved. If you imagine the left and right hand sides of this problem being balanced, like on a see-saw, then you can keep the balance by doing the same to both sides.