The teaching of division in Chaddlewood Primary School

This calculation policy outlines the progression in mathematical strategies and skills from Foundation to Year 6, and the typical year group children will be in when they are first introduced to particular concepts. It is expected that the majority of children will not draw from objectives in year groups above and below their own. Children will move towards mastery of each of the areas within their year group to ensure that they develop into confident, efficient and accurate mathematicians..

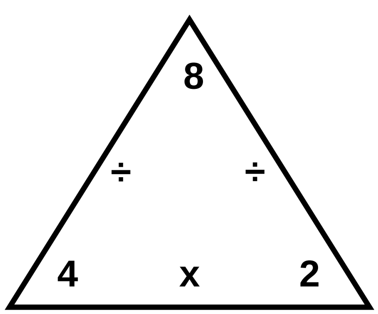
It is essential that, in all year groups, division is:
- taught alongside its inverse multiplication, as these important links will assist children in mastering the operation.
- involved in situations with rich problem solving activities and word problems.
- approached in a cross curricular manner wherever possible.

6 ÷ 3 = 2
6 ÷ 2 = 3
2 x 3 = 6
3 x 2 = 6

Children will be given many different types of problems, often which will look very different to what they are used to. This is true for all of the mathematical strategies throughout the calculation policy. For example, in calculating problems involving a missing number (for example 12 ÷ 3 = ☐), children will also consider:

☐ ÷ 3 = 4   12 ÷ ☐ = 4   4 = 12 ÷ ☐   4 = ☐ ÷ 3.

To help to develop the links between multiplication and division the children will also use ‘number trios’. Number trios demonstrate to the children that when they choose a ‘trio’, they can make four number sentences with them, by covering up particular numbers. These will be used even further by considering what would happen if we multiplied or divided each of the numbers by 10 or 100



2 x 4 = 8
4 x 2 = 8
8 ÷ 4 = 2
8 ÷ 2 = 4

Through this calculation policy, which should be read alongside our other mathematical policies, we aim for every child in our school to become;

- fluent in the strategies covered, including the rapid recall and application of key knowledge (for example times tables)

- confident and skilful at reasoning mathematically (including specialising and, eventually, generalising their conceptual understanding).

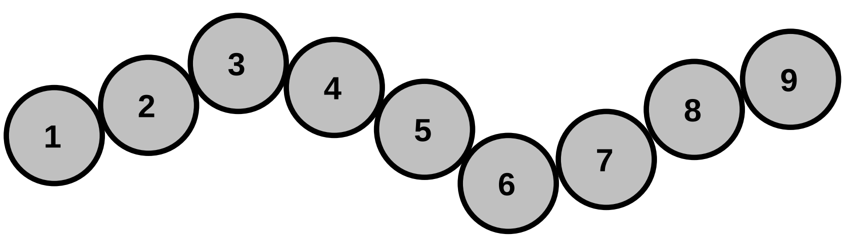
- efficient at solving problems in a sophisticated manner (for example by breaking down complex problems into simpler steps).

This calculation policy has been written to ensure a seamless progression of skills and strategies. Secondary schools in the local area have been consulted on the content contained within it, and therefore these establishments will be prepared to develop upon the knowledge and understanding that the children have when they leave our school after Year 6.

This document should also be read in conjunction with our 'models and images' policy for the appropriate operation. The policy draws upon the schools' 'Concrete, Pictorial, Abstract' approach, which emphasises the importance of mastery and the use of different representations, including through the use of conceptual variation.

Lastly, calculators will only be used when the children are secure with the appropriate strategies outlined within their year group. Calculators are a key tool in saving time with more complex problems, but they will not be used as a replacement for a thorough understanding of the underlying processes involved in calculations…

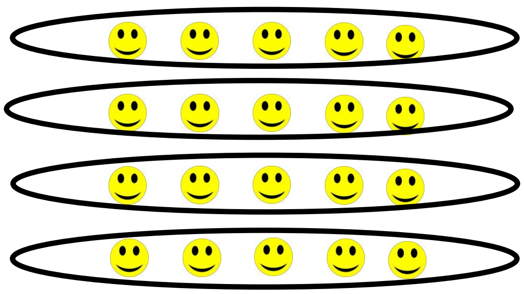
Key Vocabulary for Place Value	Key Vocabulary for Representations
Ones Tens Hundreds Thousands Ten Thousand Hundred Thousand Millions Tenths Hundredths Column Row Place holder Digit/ Integer	Bar Model Part/Part whole Dienes Base 10 Counters Cubes Bead bar/string Array Rekenrek

Strategy	*Rationale*
Practical experience of 'sharing' ***Example*** 10 fat sausages sizzling in a pan. (Encourages counting back in 2s)  ***Example*** Putting objects into pairs. 	Although division is not formally introduced until Year 1 the ground work is laid as early as the foundation stage. This includes songs that encourage jumping in equal amounts. The children will also share out toys, fruit and other materials in context. It is important that children have the opportunity to count repeatedly in groups of the same size.
Counting on in 'groups' ***Example*** Can you jump in 2s along the number track? Will you land on 7? Why not? 	Division is known as 'repeated subtraction' and it is also the opposite (inverse) of multiplication. The children will be encouraged to count forwards and backwards in 2s, 5s and 10s. **Key vocabulary** Group in pairs, threes, etc. Equal groups of

‘Grouping’

Example

20 ÷ 5 = ☐
20 children get in to teams of 5 to play a game.
How many teams are there?



There will be 4 teams.

Giving visual images for division is important.

Grouping involves the children taking a larger quantity and grouping them using a particular number (the ‘divisor’).

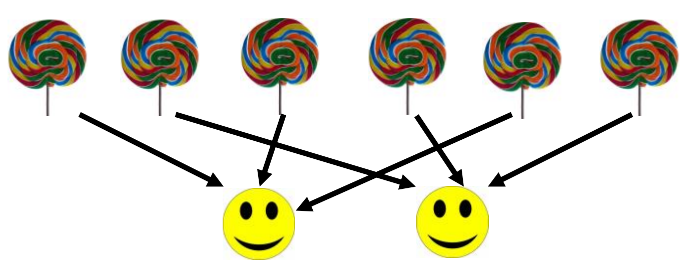
Children are encouraged to draw simple illustrations (referred to as ‘jottings’) in order to help them with problems, where they have not been supplied with a picture.

Key vocabulary
Divide, divided by, left, left over

‘Sharing’

Example

6 ÷ 2 = ☐
6 lollies are shared between 2 children.
How many lollies does each child get?



Sharing involves the children physically sharing objects (often sharing one object at a time, but this is then extended to larger amounts).

If 6 ÷ 2 = ☐ was solved by ‘grouping’ the children would think about how many groups of two there would be in 6, rather than how many each person would get.

Key vocabulary

Double, near double

Half, halve

Share, share equally

Hundred squares

Children will count on in different amounts on a hundred square, identifying patterns and answering questions associated with it.

Example

How many 5s are in 20?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Children will initially use a hundred square to explore patterns involved in counting up in 2s, 5s and 10s.

Fractions

Children will be introduced to ½ and ¼, which occur when objects cannot be shared fairly into integers ('whole' numbers).

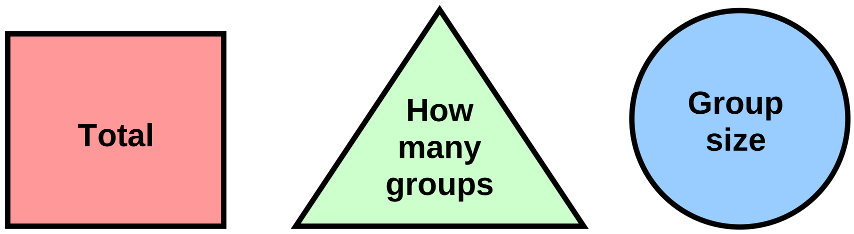
½ past can also be used with Year 1 when teaching about time (for example '½ past 5'). Emphasis should then be given to why the minute hand on the clock face is pointing downwards.

Mathematical symbols

The children will be introduced to the appropriate mathematical symbols at this stage. They should be able to write mathematical statements involving the division (÷) and equals (=) sign.

Elements of division

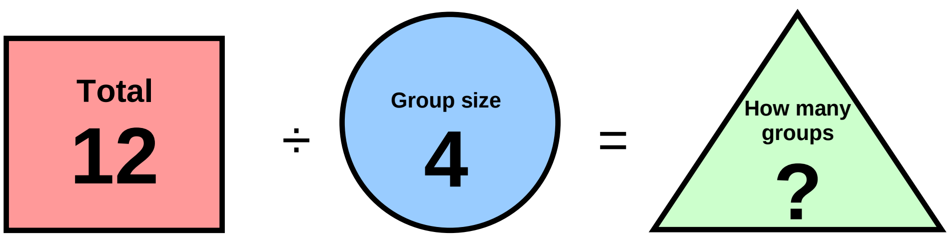
At this stage the children will be introduced to three different elements:



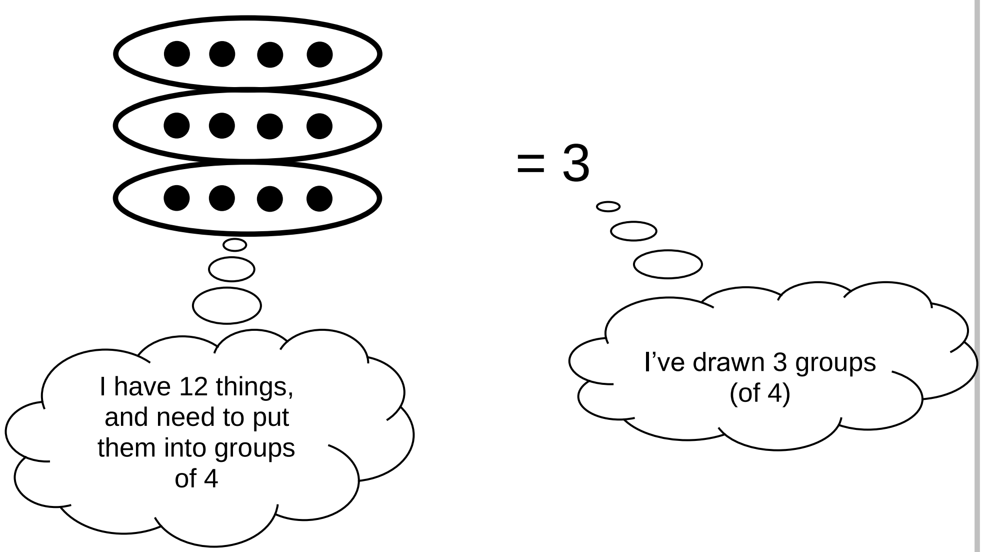
They will understand that these elements can be linked to any given division problem, in order to establish what they need to find out.

Example (as grouping)

12 ÷ 4 = ?



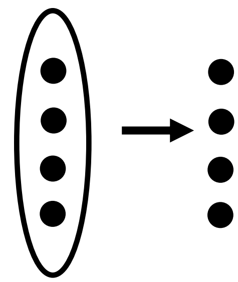
Therefore the jotting will look like the following:



The children will develop their understanding of how the numbers representing the 'total' and 'group size' can not be interchanged in this case (because division is not commutative).

In Year 2, these are linked to 'number trios' (please see above for more information regarding these)

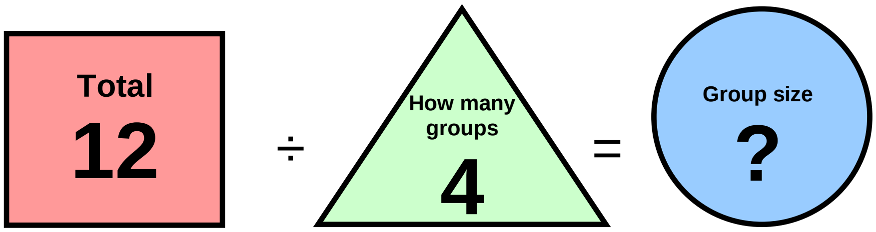
As the children become more confident with their jottings, the group 'ring' may be omitted.



Children will establish 'how many groups' by initially counting in ones up to the 'group size', and then repeating this process until all of the objects have been used. They will, however, quickly develop their ability to count in different amounts (e.g. in the particular group size, or in 2s/10s if this is more appropriate).

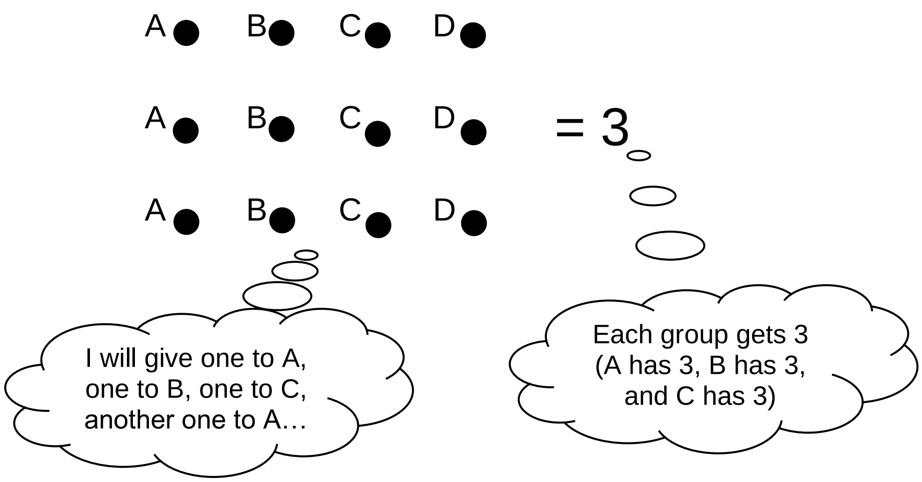
Example (as sharing)

12 ÷ 4 = ?



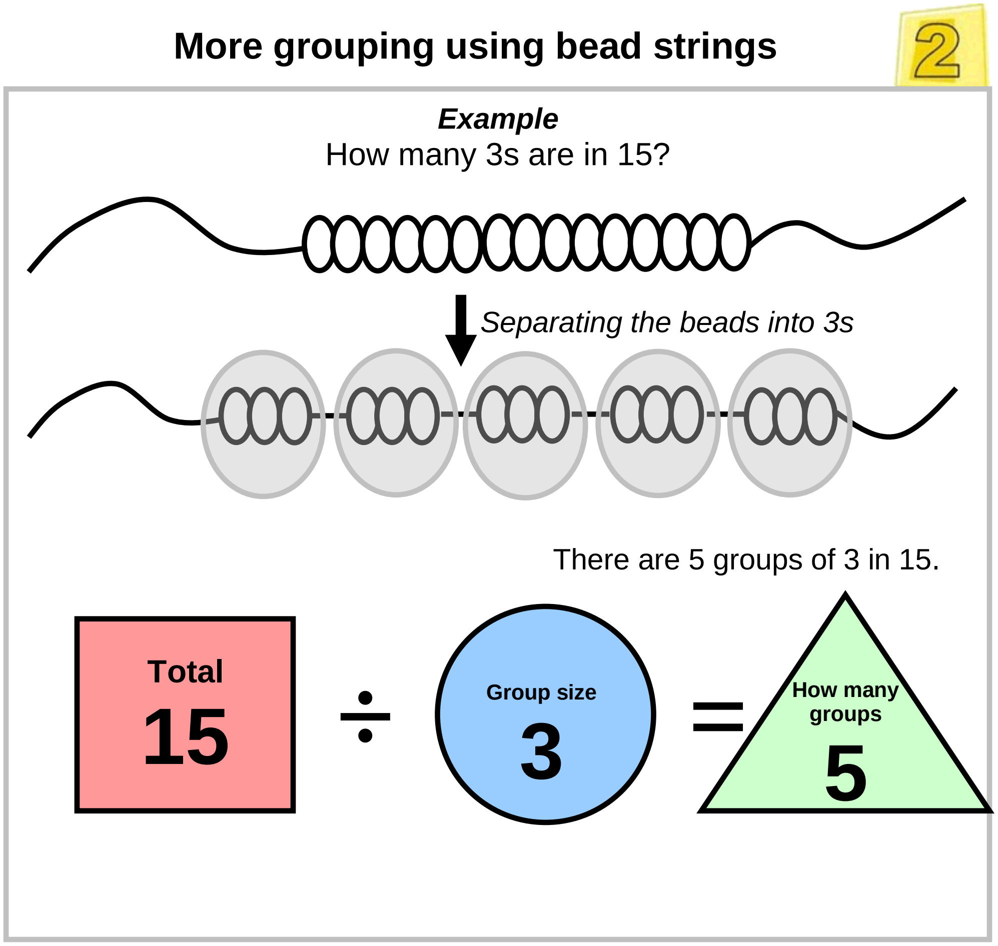
Therefore the jotting will look like the following:

There are 4 groups (A, B, C, D), sharing them to each objects looks like this:



The children will develop their understanding of how the numbers representing the ‘total’ and ‘how many groups can not be interchanged in this case (because division is not commutative).

More grouping using bead strings



Initially sharing is a powerful image for the children to use.

However, when numbers increase this can no longer be carried out practically.

It is important that when grouping is used the children make links with counting in groups using a number line.

The children are also encouraged to count up using multiplication facts (repeated addition)

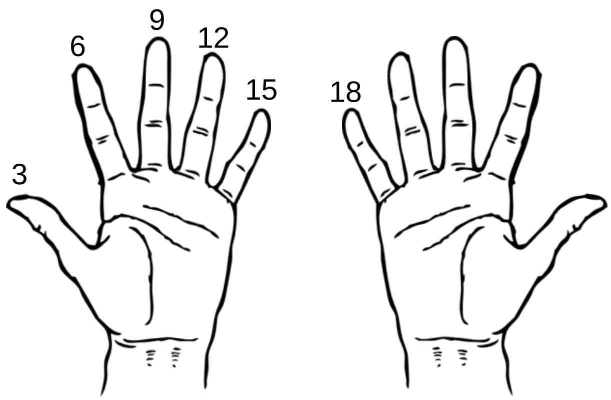
As confidence grows the children use facts to find 'missing numbers'.

Using fingers

Children count up in an amount equal to the divisor, using their fingers to help them.

Example

How many 3s are in 18?



There are six 3s in 18 (since I used six fingers)

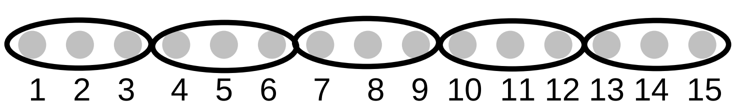
Using marked number lines

The children will combine their understanding of groupings and arrays to form an understanding of how they can use number lines to repeatedly add the divisor (and therefore find how many groups can be made).

Example

How many 3s are in 15?

15 ÷ 3 = ☐



It is important that children can make a link between grouping and jumping on a number line.

The children will also need to understand the link between division and multiplication; they are exact opposites (the 'inverse' of each other).

Fractions

Children will look at relationships of fractions,
finding halves and quarters of numbers.

Example
What is ½ of 6?
What is ¼ of 12?

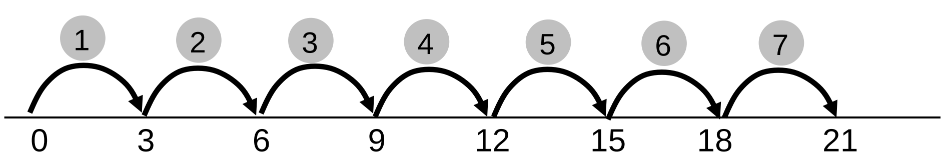
They will also recognise the equivalence of fractions, especially to ½.

Example
½ = 2/4 = 3/6 = 4/8= 5/10…

Using blank number lines

Using a number line the children will count on in an amount equal to the divisor. They will then count how many 'jumps' they made.

Example
How many 3s are in 21?



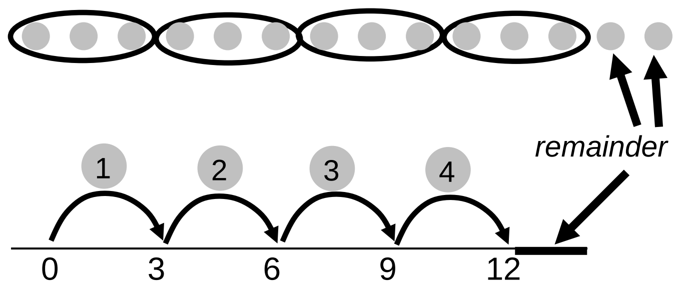
There are seven 3s in 21 (because I had to jump seven times)

Using number lines to find remainders

Children will learn how they can identify if a solution has a remainder and, if it has, what they need to do with this remainder.

Example

14 ÷ 3 = 4 remainder 2



Example

A box can hold 4 Cola bottles.

How many boxes can I fill if I have 14 bottles?

Here the answer needs to be **rounded down**.

Example

A box can hold 4 Cola bottles.

How many boxes will I need for 14 bottles?

Here the answer needs to be **rounded up**.

Children will use the language of 'remainder'.

Grouping objects and drawing number lines gives a good visual understanding of remainders.

Children will become familiar with interpreting the remainders, when faced with 'real life' problems.

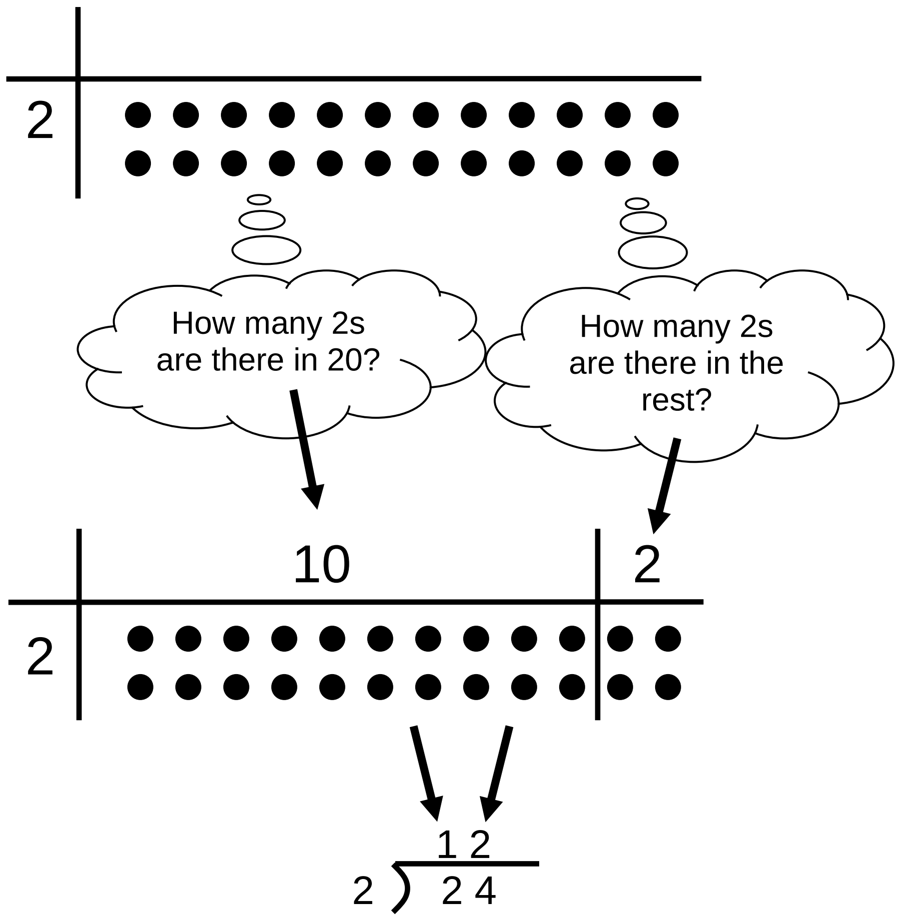
Instead of writing 'remainder', children may also abbreviate this and write 'r.'

Introduction to the 'bus shelter' method

In preparation for Year 4 and beyond, the children will begin to turn their jottings into a more formal method for division.

Example

24 ÷ 2 = ?
£24 was shared between 2 people.
How much money do they each receive?



In Year 3 children will be limited to fairly small two-digit numbers initially to ensure that their jottings are not too complex.

Once mastery of the strategy has been achieved for these smaller numbers (and a thorough understanding gained of what is happening in this process), the children will explore larger two-digit numbers.

Fractions

At this stage children will be able to;
- count in 1/10s
- recognise equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole (for example 5/7 + 1/7 = 6/7)
- order fractions from smallest to largest

Scaling

Children will solve different problems involving scaling. In doing so, they will develop their ability to apply division much more flexibly.

Example

Children will see that the shorter ribbon is a quarter of the length of the longer one, rather than 15cm shorter.

Interpreting remainders

Children will solve more complex division problems, interpreting the remainders within the solution.

Example

Cub scouts can sleep 6 to a tent. How many tents will be needed if 75 cub scouts are going camping?

Using a calculator to derive the answer gives on the display 12.5

Is the answer 12, 12.5 or 13?

Calculators are introduced as a way of solving division problems. However, the children need to interpret the displays carefully.

This builds upon the ‘Use number lines to find remainders’ method above.

Fractions

At this stage children will be able to;
- count in 1/100s (up and down)
- recognise ‘families’ of equivalent fractions
- know the decimal equivalent to ½, ¼ and ¾.
- compare numbers with the same number of decimal (up to 2 decimal places)

Division by 10, 100

The children will learn what happens to an integer when it is divided by 10 and 100. This will consolidate their place value, where they may have to express the solution in terms of ones, tenths and hundredths.

The ‘Bus Shelter’ method (1)

As the children start to use more complex methods, these continue to be supported by jottings. A formal method of division, without a number line, will be introduced.

Example

356 ÷ 8 = ☐

Quotient	Divisor) Dividend	Step
0	8) 3 5 6	How many 800s are in 300? (0)
0	8) $3\,{}^{3}5\;6$	The 300 is therefore placed into the tens column.
0 4	8) $3\,{}^{3}5\;6$	How many 80s are in 350? (4)
0 4	8) $3\,{}^{3}5\,{}^{3}6$	The remaining 30 is placed in the ones column.
0 4 4 r.4	8) $3\,{}^{3}5\,{}^{3}6$	How many 8s are in 36? (4) How many remainders?

356 ÷ 8 = 44 r. 4

Children will be encouraged to check their answers by carrying out the inverse of the operation wherever necessary

This method can not always be used, and therefore children may need to rely upon earlier strategies. For example, this method can not be used to calculate 56 ÷ 6.

Children will still need to make jottings to support them with this method. For example, quickly writing down all multiples of the divisor (in this case 8s) down the side of the page will help to efficient calculate each element of the final solution.

Key vocabulary

Formal written method

Dividend, divisor, short division

Bus shelter

The 'Bus Shelter' method (2) Children will then extend upon this method using; - a two / three digit divisor, - non-integer numbers (maintaining the decimal point in their calculations) using jottings once again to support them in both of these situations.	The children will continue to interpret remainders appropriately, taking into consideration the original context of the problem.
Division by 10, 100 and 1000 The children will learn what happens to a number when it is divided by 10, 100 and 1000. These numbers will now also include decimal numbers.	
Fractions At this stage children will be able to; - compare fractions confidently (where their denominators are all multiples of the same number) - recognise mixed numbers and improper fractions, and to be able to convert between them (for example 2/5 + 4/5 = 6/5 = 1 1/5 - read and write decimal numbers as fractions - read, write, order and compare numbers (up to three decimal places)	

Brackets

Children will develop an understanding of how to approach problems which involve the use of brackets, including the mathematical rules underpinning extended number sentences (for example that they should always solve the mathematical calculation within the brackets first, and be able to read problems where mathematical symbols have been omitted).

Children will be introduced to the term 'BODMAS' to represent the order that operations need to be carried out.

Brackets
Order (for example, 'powers' such as 3^2)
Division
Multiplication
Addition
Subtraction

Notice that, in the first example below, the solution is different depending upon where the brackets are placed.

Example

(20 ÷ 4) + 1 = (5) + 1 = 6

20 + (4 + 1) = 20 ÷ (5) = 4

Example

3(7+2) = 3(9) = 27

In these examples brackets are used to increase the complexity of the calculation.

Key vocabulary
Order of operations

Long division

Children will continue to develop their ability to divide larger numbers, using jottings to support them. 'Long division' will be introduced to help when the divisor is at least a two-digit number. Children will use this method with <u>up to 4 digit numbers</u>.

Example

432 ÷ 15 = ☐

```
        2 8  r 12
      ________
15  ) 4 3 2
       3 0 0      (15x20)
      ________
       1 3 2
     - 1 2 0      (15x8)
      ________
         1 2
```

15 x 10= 150
15 x 20= 300
15 x 9= 135
15 x 8= 120

432 ÷ 15 = 28 r.12

…or, because 12/15 = 4/5…

432 ÷ 15 = 28 4/5

…or….

```
        2 8 . 8
      ________
15  ) 4 3 2
       3 0 0      (15x20)
      ________
       1 3 2
     - 1 2 0      (15x8)
      ________
         1 2 . 0
```

15 x 10= 150
15 x 20= 300
15 x 9= 135
15 x 8= 120
15 x 0.8= 12

Multi-step problems and decisions about:
(i) which operation to use
(ii) the degree of accuracy in each calculation

At this stage the children will be proficient at being able to identify when they are required to divide a set of quantities.

The children will build upon the vocabulary work that they have previously experienced. Mastery of this concept will also be developed through the use of visually representing particular problems (through the use of jottings).

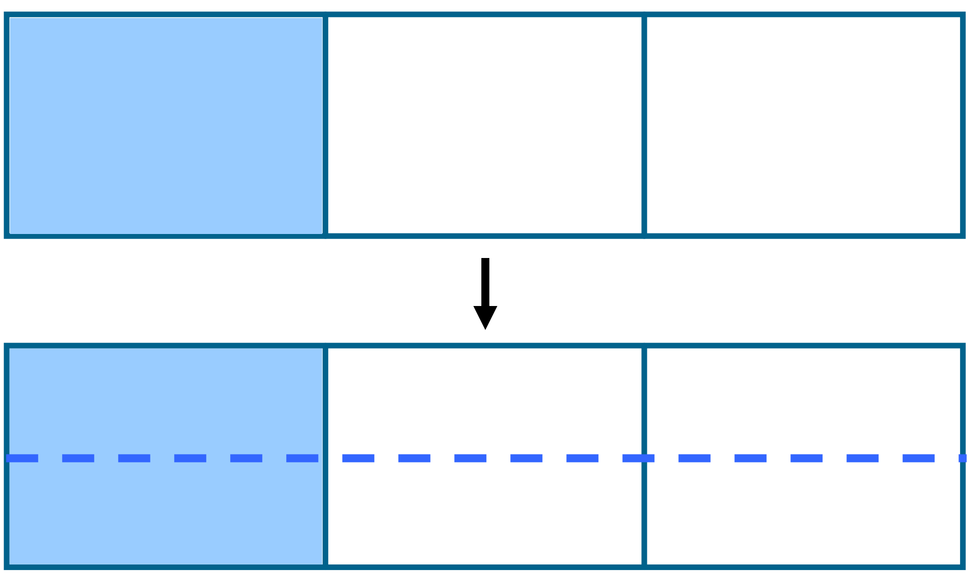
It the children do not need to divide in a calculation, they will still be able to identify which other operation to use.

Division of fractions and mixed numbers

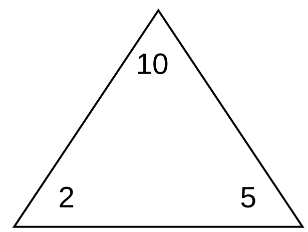
Example

$$\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$$

because….



When dividing 'families' or numbers, number trios will be used to identify the relationship between all of them.



1/5 x 1/2 = 1/10
1/2 x 1/5 = 1/10
(and 10 x 1/2 = 5)
(and 10 x 1/5 = 2)
1/10 ÷ 1/2 = 1/5
1/10 ÷ 1/5 = 1/2

Long division (2)

Children will continue to develop their ability to divide larger numbers, using their knowledge of short division. 'Long division' will be introduced to help when the divisor is at least a two-digit number. Children will use this method with up to 4 digit numbers.

Example

432 ÷ 15 = ☐

```
       2 8   r 12
     ___________
23  ) 1 2 3
       3 0 0        (15x20)
     ___________
        1 3 2
      - 1 2 0       (15x8)
     ___________
           1 2
```

15 x 10= 150
15 x 20= 300
15 x 9= 135
15 x 8= 120

432 ÷ 15 = 28 r.12

…or, because 12/15 = 4/5…
432 ÷ 15 = 28 4/5

…or….

```
       2 8  . 8
     ___________
15  ) 4 3 2
       3 0 0        (15x20)
     ___________
        1 3 2
      - 1 2 0       (15x8)
     ___________
           1 2 . 0
```

15 x 10= 150
15 x 20= 300
15 x 9= 135
15 x 8= 120
15 x 0.8= 12

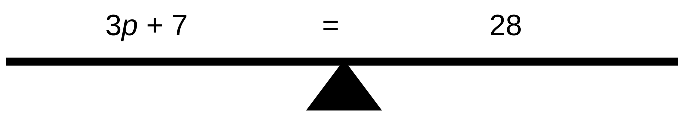
Algebra

The children will learn that algebra involves the use of simplified number sentences, where both sides of the equals sign needs to 'balance'.

Example

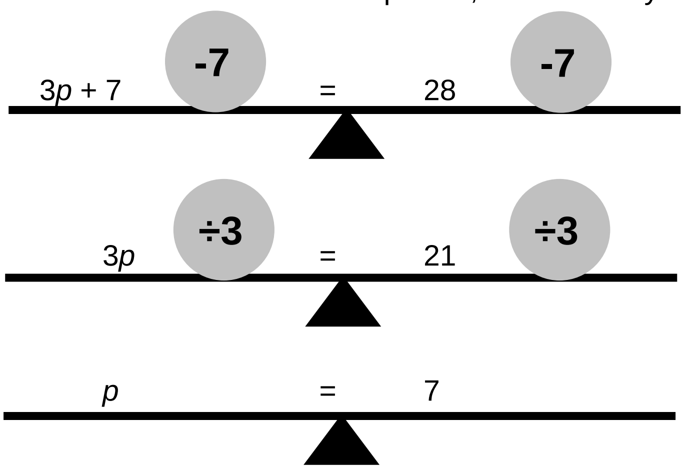
$3p + 7 = 28$

In this case *p* represents a missing number, but any letter could be used. Imagine the number sentence, balanced on a see-saw.



To keep the see-saw balanced whatever happens to one side must happen to the other. As we need to find the value of *p* our goal is to isolate this letter, with all of the numbers are on the other side of the equation.

Subtract 7 from both sides of the equation, and divide by 3.



Algebra may look confusing, but it is simply a way of representing a missing number with a letter.

The children have tackled problems similar to this much earlier in their school lives, for example in 'missing number' sentences. The missing number boxes are now just replaced with a mathematical symbol.

$\square + 6 = 15$

$f + 6 = 15$

Extended algebra

Example

$4f + 7 = f + 16$

$4f + 7 - 7 = f + 16 - 7$ (subtract 7 from both sides)

$4f = f + 9$

$4f - f = f + 9 - f$ (subtract *f* from both sides

$3f = 9$

$f = 3$ (divide both sides by 3)

This example is more involved. If you imagine the left and right hand sides of this problem being balanced, like on a see-saw, then you can keep the balance by doing the same to both sides.