The teaching of addition in Chaddlewood Primary School

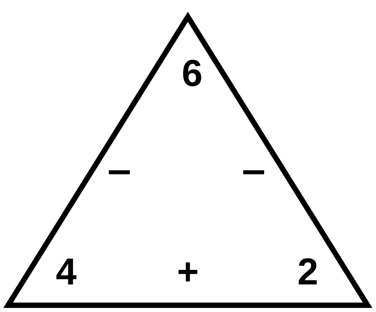
This calculation policy outlines the progression in mathematical strategies and skills from Foundation to Year 6, and the typical year group children will be in when they are first introduced to particular concepts. It is expected that the majority of children will not draw from objectives in year groups above and below their own. Children will move towards mastery of each of the areas within their year group to ensure that they develop into confident, efficient and accurate mathematicians.

It is essential that, in all year groups, addition is:
- taught alongside its inverse subtraction, as these important links will assist children in mastering the operation.
- involved in situations with rich problem solving activities and word problems.
- approached in a cross curricular manner wherever possible.

Children will also be given many different types of problems, often which will look very different to what they are used to. This is true for all of the mathematical strategies throughout the calculation policy. For example, in calculating problems involving a missing number (for example 10 + 3 = ☐), children will also consider:
☐ + 3 = 13 10 + ☐ = 13 13 = 10 + ☐ 13 = ☐ + 3.

2 + 7 = 9
7 + 2 = 9
9 – 7 = 2
9 – 2 = 7

To help to develop the links between addition and subtraction the children will also use ‘number trios’. Number trios demonstrate to the children that when they choose a ‘trio’, they can make four number sentences with them, by covering up particular numbers. These will be used even further by considering what would happen if we multiplied or divided each of the numbers by 10 or 100.



2 + 4 = 6
4 + 2 = 6
6 – 2 = 4
6 – 4 = 2

Through this calculation policy, which should be read alongside our other mathematical policies, we aim for every child in our school to become;

- fluent in the strategies covered, including the rapid recall and application of key knowledge (for example times tables)

- confident and skilful at reasoning mathematically (including specialising and, eventually, generalising their conceptual understanding).

- efficient at solving problems in a sophisticated manner (for example by breaking down complex problems into simpler steps).

This calculation policy has been written to ensure a seamless progression of skills and strategies. Secondary schools in the local area have been consulted on the content contained within it, and therefore these establishments will be prepared to develop upon the knowledge and understanding that the children have when they leave our school after Year 6.

This document should also be read in conjunction with our ‘models and images’ policy for the appropriate operation. The policy draws upon the schools’ ‘Concrete, Pictorial, Abstract’ approach, which emphasises the importance of mastery and the use of different representations, including through the use of conceptual variation.

Lastly, calculators will only be used when the children are secure with the appropriate strategies outlined within their year group. Calculators are a key tool in saving time with more complex problems, but they will not be used as a replacement for a thorough understanding of the underlying processes involved in calculations.

Key Vocabulary for Place Value	Key Vocabulary for Representations
Ones Tens Hundreds Thousands Ten Thousand Hundred Thousand Millions Tenths Hundredths Column Row Place holder Digit/ Integer	Bar Model Part/Part whole Dienes Base 10 Counters Cubes Bead bar/string Rekenrek

<table>
<tr><th>Strategy</th><th>Rationale</th></tr>
<tr><td>
Using songs and number rhymes

Teachers will use common songs and number rhymes to build up an understanding of pattern and vocabulary, and to develop fine motor skills.

Addition vocabulary is built in.
This includes ‘add’, finding ‘one more’ than a quantity, and establishing which quantity is ‘more than’ another.
</td><td>
Children use their counting skills to find one more than a quantity, using their fingers to help them to count (from 10)

They will use objects, pictures, stories and songs to help develop their understanding.

At this stage the children will count and point using objects, whilst physically moving them. Whenever possible we will use real life experiences to develop their understanding of addition, and how it relates to subtraction.

Children will need lots of these experiences before moving onto more abstract forms of addition.
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Combining sets of objects

Children will count out a quantity, and will then be asked ‘How many do you have altogether?

Example
Count out 3 strawberries.
Count out 2 strawberries.

How many strawberries altogether?

Example
At a party I eat 2 cakes and my friend eats 3.
How many cakes did we eat altogether?

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Early addition is about combining 2 sets of objects physically. This may include the use of fingers.

As children become confident they combine the numbers to find an answer without using physical apparatus and objects, and by increasingly being able to manipulate numbers mentally.
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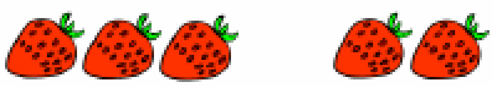
These physical representations are then linked to number sentences.

Example	*Example*
 3 + 2 = 5	 2 + 3 = 5

The children will be introduced to the appropriate mathematical symbols at this stage. They should be able to read, write and interpret mathematical statements involving the addition (+) and equals (=) sign.

Children will also understand the effect that 'adding zero' has upon a quantity.

Using a number line

Children will be asked to solve addition calculations with totals of less than 20. In doing so in Year 1 they should be confident in their use of number bonds within 20.

The children will therefore start to develop an understanding of 2 digit numbers, and what these represent.

Initially, children use a marked number line to calculate addition problems.

Example

3 + 2 = 5 (You start on 3, and 'jump on 2')

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Example

7 + 6 = 13 (You start on 7, and 'jump on 6')

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Children will start to gain an understanding of 2 digit numbers, and why it is more efficient to start on the higher numbers before 'jumping on'.

Children are encouraged to use a large number line, and to count on in ones (often using a finger or pen to mark each jump). Initially this method would be used alongside previous methods until the children are confident in using a number line.

At this stage the children will develop their ability to solve 'missing number' statements, such as
☐ + 3 = 13 and
10 + ☐ = 13

Key vocabulary

Number bonds, number line

Add, more, plus, make, sum, total, altogether, combine

Equals, is the same as (including equals sign)

Inverse

Introducing a hundred square

Children will move to using a hundred square to 'jump on'. They will initially start on the larger number, and then jump on.

Example

5 count on 2 = 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5 + 2 = 7

They will then move to a more efficient method of adding 10 to a number (jumping vertically rather than horizontally).

Example

38 count on 10 = 48

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

38 + 10 = 48

Children begin to use 100 squares as a tool to aid counting on in small steps (eg. in 1s or 2s)

Once secure they begin to use the 100 square to count on in tens.

Children learn that, as they move down a row, they add on 10 each time.

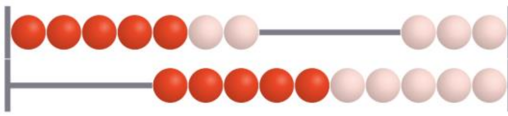
Careful attention is given to possible misconceptions at this stage, especially 'jumping on' their starting number (instead of always moving horizontally with each move).

Using rekenrek

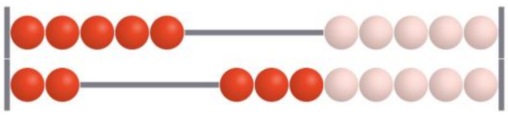
The children will use rekenrek as a key tool to compose and decompose different numbers.

They will be able to represent numbers in varying ways using this frame.

Example
5 + 2 = 7



or



The children will learn how to describe the image to the left using stem sentences such as

‘7 is the whole. 5 is a part and 2 is a part’

and

‘5 needs 2 to make 7’.

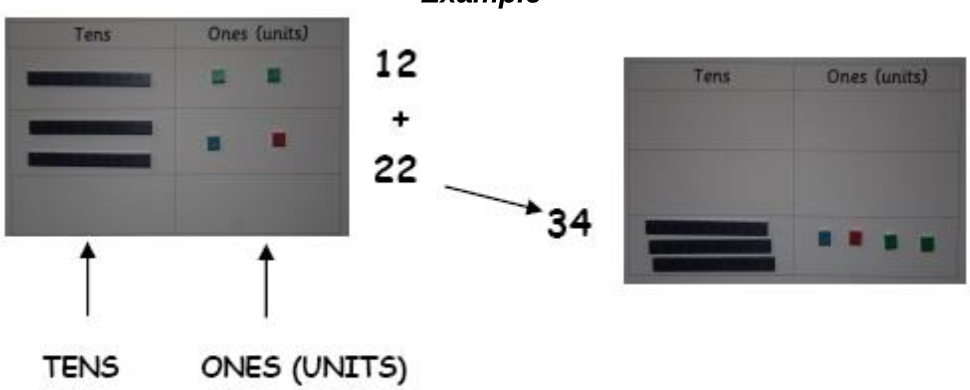
Introducing partitioning (2 digit numbers)

Children will learn that numbers 10 or over (and under 100) are made up of TENS (left hand digit) and ONES (right hand digit).

Partitioning a number involves splitting it up into TENS/ONES to show the value of each digit.

Numbers can then be added by first combining the TENS and then combining the ONES.

Example



Initially this will be practically done using ‘Deines’ (which comprise of sticks representing ‘tens’, and cubes representing ‘ones’).

This method is also used when children are introduced to the idea of adding HUNDREDS.

As children become secure they will say the value of each digit without apparatus.

Key vocabulary
Partition

More complex addition using a hundred square

*Prior to using the hundred squares below the children will need to have a secure understanding of the value of each digit in a number, as determined by its position. This is called **place value**.*

Extension of vocabulary associated with addition

By this stage the children should have developed their understanding of different vocabulary associated with addition, including:

Put together, add(ition), altogether, total, distance between, more than

This will be developed, in conjunction with the subtraction calculation policy, to also include:
Take away, subtract(ion), difference between, less than

Using the hundred square to combine 2 digit numbers

Example

$47 + 12 = \square$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Adding 12 involves moving down a row, and then to the right 2 places.

Children learn to use the hundred square to add 2 digit numbers.

In this example the children will understand that 'one ten' (10) and 'two ones' (2) is added to 47. They will therefore move down a row on the 100 square to add 10, and then move two spaces to the right to add 2.

Children will, once again, consider the inverse of addition in doing so.

Children will also use:
- their addition facts up to 20 for simpler calculations (they will therefore recognise when they do not need to use a hundred square)
- derive facts up to 100, such as 60+30, which will help to minimise the steps involved in this strategy

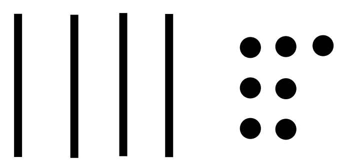
Addition using jottings

The efficiency of the above method is then improved even further. The children will be encouraged to represent their workings using a series of jottings showing 'sticks' (to represent tens) and 'dots' (to represent ones).

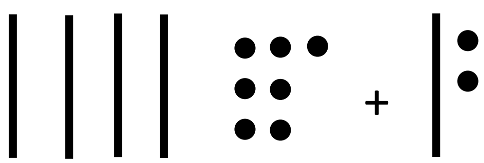
Example

47 + 12 = □

"47 is made up of 4 TENS and 7 ONES"



"I need to add 12. That's 1 TEN and 2 ONES"

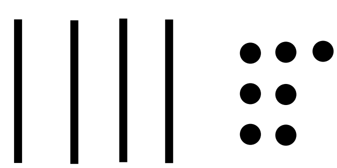


"I will now count how many I have (starting with counting the 'sticks' of ten).
10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59… so the answer is 59"

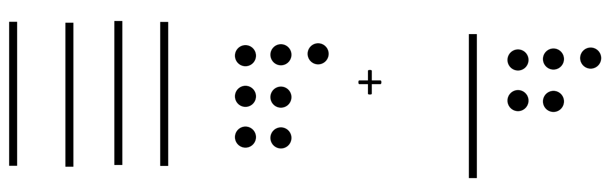
Example

47 + 15 = □

"47 is made up of 4 TENS and 7 ONES"



"I need to add 12. That's 1 TEN and 5 ONES"



"I will now count how many I have (starting with counting the 'sticks' of ten).
10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63…
so the answer is 63"

Children initially draw the appropriate number of 'sticks' and 'dots' to represent their starting number. They then jot the second number.

The children are encouraged to firstly count the TENS, and then to count up in the appropriate number of ONES (in case the number breaches a multiple of ten).

Whilst not essential at this stage, children can also be introduced to drawing a small 'square' to represent the HUNDREDS block (as this is used whilst using Deines). This can then be used to add and subtract three-digit numbers.

Children who master the example of the left so be encouraged to recombine 10 ONES into a single TEN to simplify their jottings and to minimise any counting mistakes.

Marked number lines

The children will use number lines to support them in their calculations, whilst consolidating their mental strategies.

This will initially involve a marked number line, with the children jumping on in an appropriate number of tens and ones.

Example

23 + 12 = □

+10 +2

23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

23 + 12 = 35

The children will also use number lines to add three one-digit numbers together accurately (using their known addition facts to combine number bonds wherever possible).

Example

4 + 9 + 7 = □

I know that it is easier to start on the 'biggest' number when adding...

+4 +7

7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

4 + 9 + 7 = 20

Example

6 + 7 + 4 = □

I know what 6+4 is!

6+4 = 10

This means that I have to calculate 10+7…

…and I know that 1 'ten' and 7 'ones' is 17.

6 + 7 + 4 = 17

It is important for children here to appreciate that number lines go on infinitely, including into negative numbers.

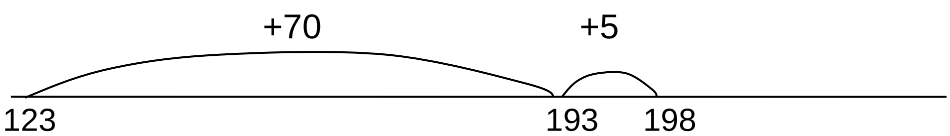
At this stage, Deines may once again be used to show the children the effect of adding 'tens' and 'ones' to a number (although care needs to be taken when the amount of 'ones' exceed 9 so that the answer is recorded correctly).

Unmarked number lines

The children will move towards adding larger multiples of tens and ones, using unmarked ('empty') number lines.

Example

123 + 75 = ☐



123 + 75 = 198

Children will begin to use 'empty' number lines - starting with the largest number and counting on in tens and then ones.

Breaching ten

Children will continue to develop their mastery of the above number line and hundred square strategies, moving towards breaching multiples of tens.

Introduction of the formal method for addition

The children will be introduced to more formal methods, building on their ability to partition effectively. They will initially add together 2 two-digit numbers using a formal method for addition.

Example

34 + 13 = ?

```
  34
+ 13
____
  47
____
```

Example

34 + 18 = ?

```
  34
+ 18
  1
  52
```

so 34 + 18 = 52

Children will be taught written methods for those calculations they cannot do ‘in their heads’.

These methods encourage pupils to think about the value of digits, but the addition of the numbers is still done mentally.

Children are introduced to column addition without any breaching of ten.

Once this has been mastered, the children move to examples where the ONES total more than ten. In these instances the children record the ‘ten’ on a blank line that has been intentionally left, above the first horizontal line, in the TENS column.

Key vocabulary
Column addition

Formal written method for addition

The above methods are then condensed further, including problems which require 'carrying'.

Children in Year 3 will use this method with numbers up to 1,000.
Children in Year 4 will use this method with numbers up to 10,000.
Children in Year 5 will use this method with numbers up to, and beyond, 10,000.

Example

In the below example you add the ones; 7+5=12. There are 2 ones in '12', and 1 ten, hence 2 in the 'ones' column, and 1 in the 'tens' column.

```
 587
+375
  1
   2
```

The next stage is to look at the 'tens' column, ensuring that place value is maintained (e.g. verbally stating 80+70, rather than 8+7).

```
 587
+375
 11
  62
```

Continue until all columns have been totalled.

```
 587
+375
 11
 962
```

This column addition follows the more conventional method must adults are familiar with.

When the total number of ones in two numbers exceed 9, one ten is carried across to the tens column. This is known as 'carrying'.

'Carried numbers' should always be written above the first horizontal line, in the appropriate column. Children are then continually encouraged to remember to include these when totalling the specific column.

Children are shown to start on the <u>least significant number</u> (the 'ones') and add the columns from right to left.

Key vocabulary

Efficient written method

Predicting solutions and checking their suitability.

Through the use of the above strategy, children will ensure that their solutions are ‘reasonable’.

Children will estimate, before calculating, what their solution is likely to be close to. This will involve increasingly precise rounding of numbers, especially in Years 5 and 6, to define the likely boundaries for their solution.

Children will also use the inverse of addition (subtraction) to check when they have doubts about the accuracy of their answer

Example

$$\begin{array}{r} 587 \\ \underline{+375} \\ \underline{} \end{array}$$

“I know that 600+400 is 1,000 and so my answer is going to be less than this,”

moving to…

“I know that 590+370 = 960, and so my answer will be close to this number.”

Example

$$\begin{array}{r} 587 \\ +375 \\ \underline{11} \\ \underline{962} \end{array}$$

“So 962 – 375 = 587”

$$\begin{array}{r} {}^{8}\cancel{9}\,{}^{1\,5}\cancel{6}\,{}^{1}2 \\ \underline{-\ 375} \\ \underline{587} \end{array}$$

Concise method for adding decimals

Example

123.9
+ 7.25
11
131.15

6.72
+ 8.56
11
15.28

Once secure with the previous methods the children will be introduced to adding decimals to decimals (and decimals to whole numbers), ensuring that place value is maintained throughout.

Children may, at this stage, record a ‘0’ in any spare space in order to assist them in maintaining place value.

Brackets

Children will develop an understanding of how to approach problems which involve the use of brackets, including the mathematical rules underpinning extended number sentences (for example that they should always solve the mathematical calculation within the brackets first, and be able to read problems where mathematical symbols have been omitted).

Children will be introduced to the term ‘BODMAS’ to represent the order that operations need to be carried out.

Brackets
Order (for example, ‘powers’ such as 3^2)
Division
Multiplication
Addition
Subtraction

Notice that, in the first example below, the solution is different depending upon where the brackets are placed.

Example

(7 + 2) x 3 = (9) x 3 = 27

7 + (2 x 3) = 7 + (6) = 13

Example

3(7+2) = 3(9) = 27

In these examples brackets are used to increase the complexity of the calculation.

Key vocabulary
Order of operations

Multi-step problems and decisions about: (i) which operation to use (ii) the degree of accuracy in each calculation At this stage the children will be proficient at being able to identify when they are required to add a set of quantities. They will be able to recognise elements to add, even in problems which involve multiple 'steps'. The children will build upon the vocabulary work that they have previously experienced. Mastery of this concept will also be developed through the use of visually representing particular problems (through the use of jottings).	It the children do not need to add in a calculation, they will still be able to identify which other operation to use.

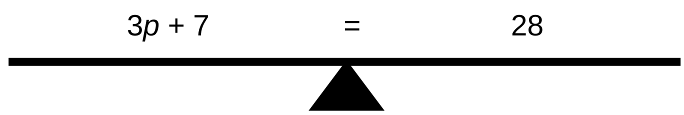
Algebra

The children will learn that algebra involves the use of simplified number sentences, where both sides of the equals sign needs to ‘balance’.

Example

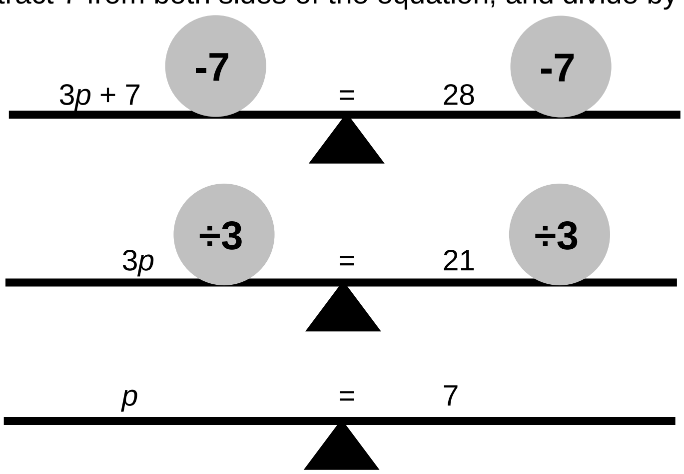
$3p + 7 = 28$

In this case p represents a missing number, but any letter could be used. Imagine the number sentence, balanced on a see-saw.



To keep the see-saw balanced whatever happens to one side must happen to the other. As we need to find the value of p our goal is to isolate this letter, with all of the numbers are on the other side of the equation.

Subtract 7 from both sides of the equation, and divide by 3.



Algebra may look confusing, but it is simply a way of representing a missing number with a letter.

The children have tackled problems similar to this much earlier in their school lives, for example in ‘missing number’ sentences. The missing number boxes are now just replaced with a mathematical symbol.

$\square + 6 = 15$

$f + 6 = 15$

Extended algebra

Example

$4f + 7 = f + 16$	
$4f + 7 – 7 = f + 16 – 7$	(subtract 7 from both sides)
$4f = f + 9$	
$4f – f = f + 9 – f$	(subtract f from both sides
$3f = 9$	
$f = 3$	(divide both sides by 3)

This example is more involved. If you imagine the left and right hand sides of this problem being balanced, like on a see-saw, then you can keep the balance by doing the same to both sides.