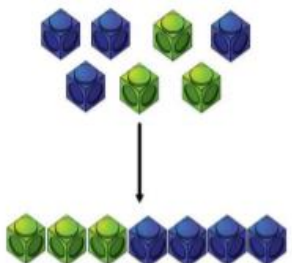
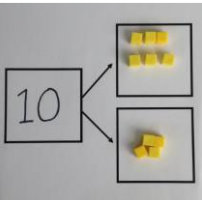
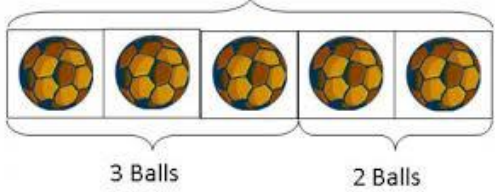
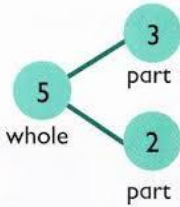
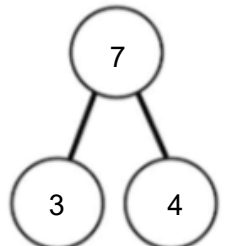


The teaching of addition in Chaddlewood Primary School

Progression of models

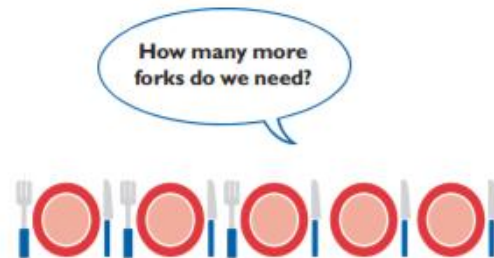


This policy outlines the progression in mathematical models and images from Foundation to Year 6. The policy draws upon the schools' 'Concrete, Pictorial, Abstract' approach, which emphasises the importance of mastery and the use of different representations, including through the use of conceptual variation. This policy should also be read in conjunction with the relevant calculation policy.

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-part-whole model	Combine a mixture of cubes into coloured groups.  Use cubes to add two numbers together as a group or in a bar.  	Use pictures to add two numbers together as a group or in a bar.   	Develop the 'part-part-whole' representation using numerals.  

Combining two parts to make a whole: part-part-whole model (cont.)

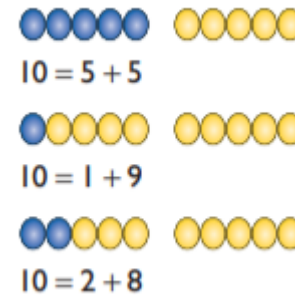
Using objects, such as in a role play area, identify 'how many more' and 'how many altogether'.



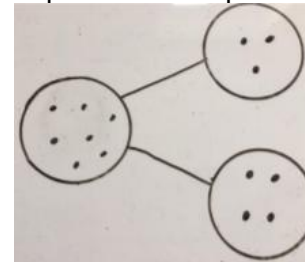
$$20 = 12 + 8$$

"There are 20 people on this bus. 12 are on the bottom and 8 are on the top."

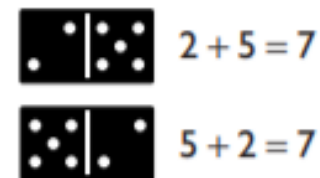
Partition and recombine the 'total' using pictures.



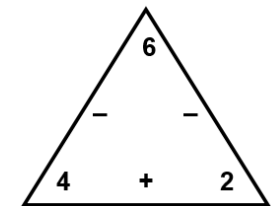
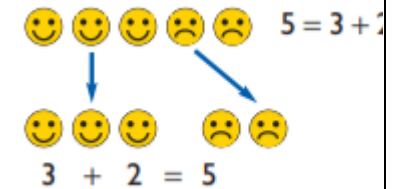
Introduce the 'part-part-whole' representation using dots.



Notice how addition is commutative.



Link part-part-whole model with number trios.



Record mathematically using numerals.

$$4 + 3 = 7$$

$$10 = 6 + 4$$

Starting at the larger number and counting on

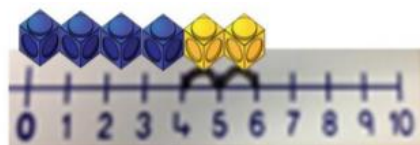
With physical objects, start by counting the larger quantity and then 'count on' the smaller.



Start with the larger number on a bead string and then 'count on' the smaller.



Count on using number lines and cubes.



Count on using a hundred square.
5 count on 2 = 7

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use a number line and a decreasing number of intervals (move towards a blank number line).

Count on in ones, or in one complete jump.

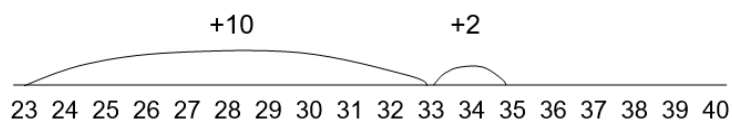
Identify how it is more efficient to start on the larger number.

2 count on 5

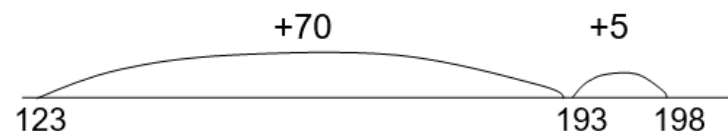


5 count on 2

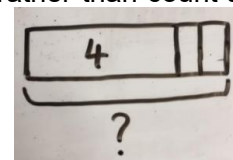
$$23 + 12 = 35$$



$$123 + 75 = 198$$



Use a bar model to encourage the children to count on, rather than count all.



Record mathematically using numerals.

$$5 + 12 = 17$$

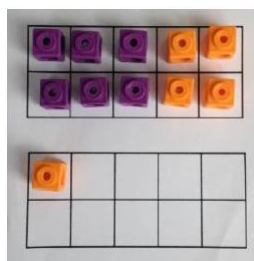
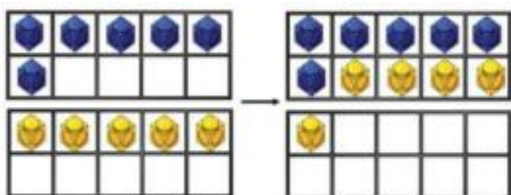
Starting at the larger number and counting on (cont.)

$$47 + 12 = \square$$

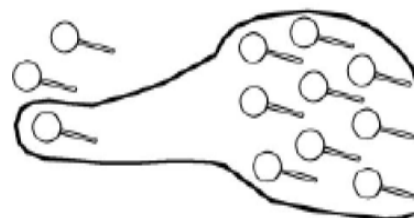
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Regrouping to make 10.

Start with the larger number and use the smaller number to make 10 (for example. 9+3 is regrouped to 10).

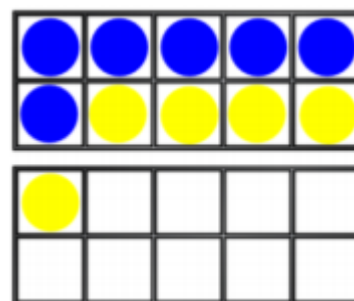


Using a jotting, circle 10 items.



$$3 + 9 =$$

Draw ten frames and arrange differently coloured dots to make 10.



Using a number line, regroup or partition the smaller number to make 10.

Record mathematically using numerals.

$$7 + 4 = 11$$

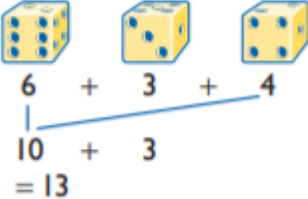
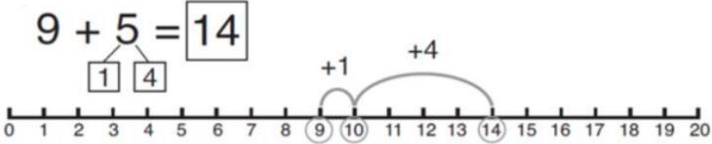
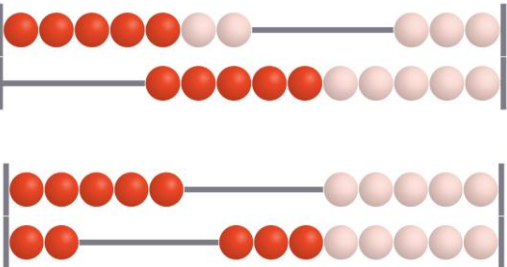
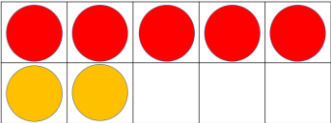
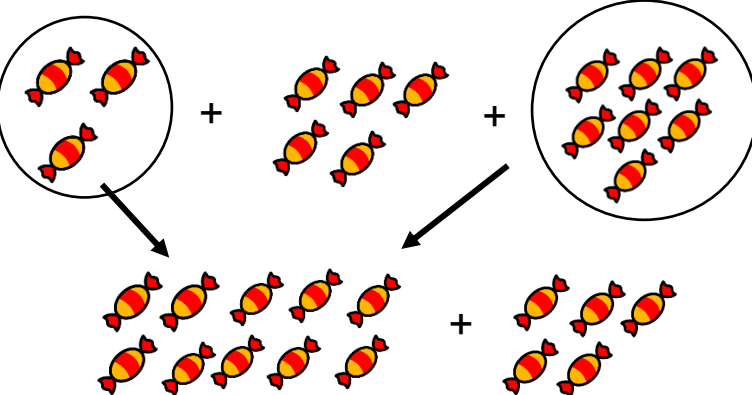
$$4 + 7 = 11$$

Develop an understanding of equality through simple mathematical equations.

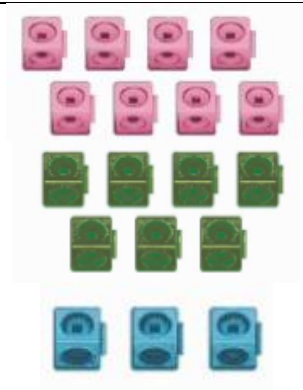
$$6 + ? = 11$$

$$6 + 5 = 5 + ?$$

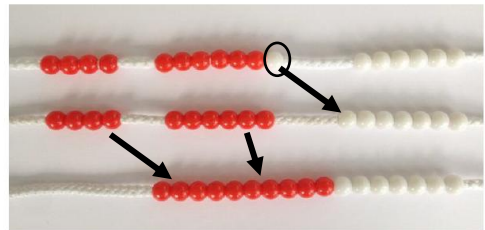
$$6 + 5 = ? + 4$$

Regrouping to make 10 (cont.)	Notice number bonds to 10 using dice. 		
Making other amounts	Using rekenrek and stem sentences such as • '7 is the whole. 5 is a part and 2 is a part'. • '5 needs 2 to make 7'. This is represented by either of the following rekenrek images. 	Using two different coloured dots on a tens frame. 	Record mathematically using numerals. $5 + 2 = 7$ $7 = 5 + 2$ Develop an understanding of equality through simple mathematical equations. $5 + ? = 7$ $2 + 5 = 5 + ?$
Adding three single digits	Using objects, add together starting with the larger quantity.	Add together three groups of objects. Draw a picture to recombine the groups to make 10 if possible. 	Combine two numbers to make 10 (if possible) and then add on the final number. Record mathematically using numerals.

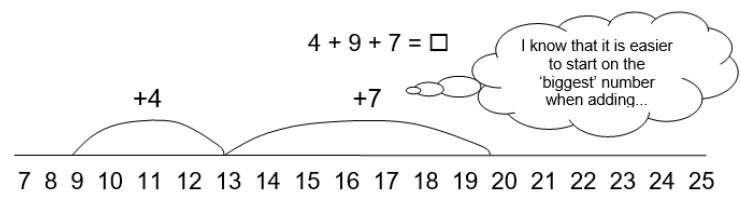
Adding three single digits (cont.)



Spot any number bonds to 10 and then calculate the total.



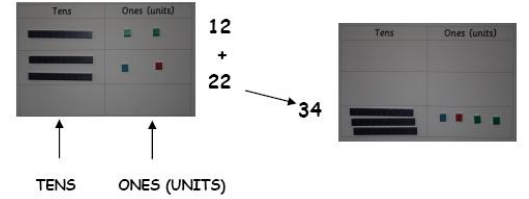
Add three single digits using a number line.



$$\begin{aligned} 4 + 7 + 6 &= 10 + 7 \\ &= 17 \end{aligned}$$

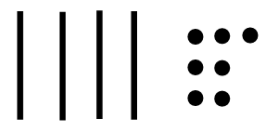
Column method - no regrouping

Using base 10/Deinnes blocks and place value counters, partition numbers appropriately and add to find the total. Add together the ones/units first then add the tens.

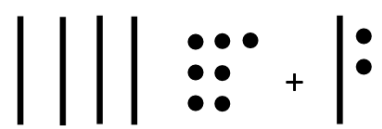


Represent the base 10/Deinnes blocks using lines for tens and dots for ones.

"47 is made up of 4 TENS and 7 ONES"

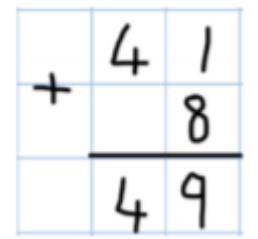


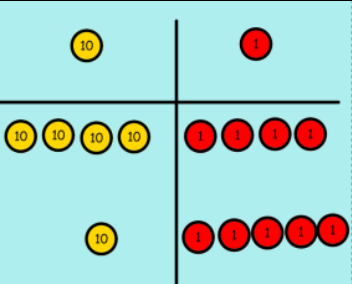
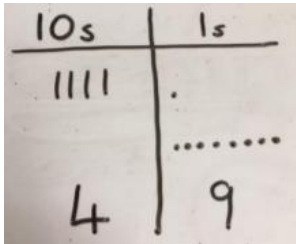
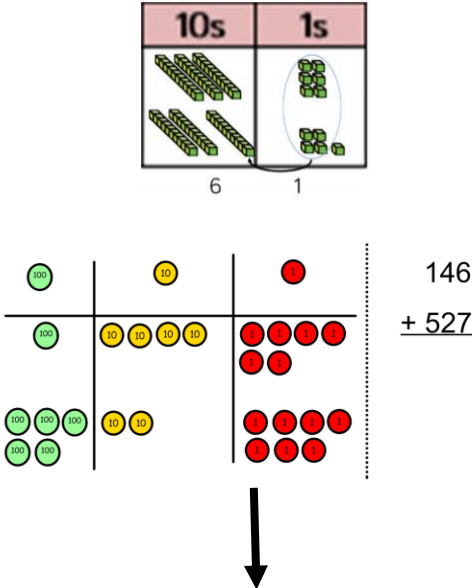
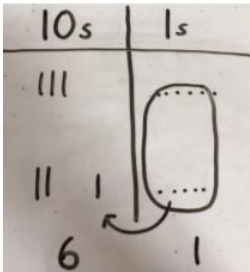
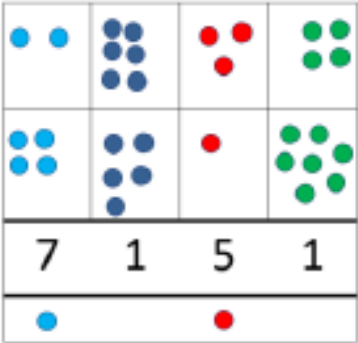
"I need to add 12. That's 1 TEN and 2 ONES"



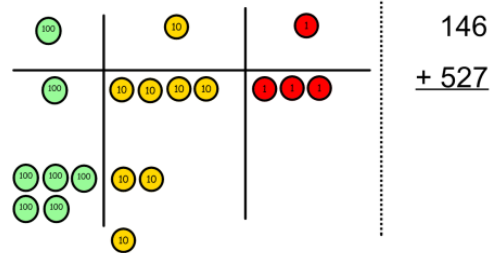
"I will now count how many I have (starting with counting the 'sticks' of ten). 10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59... so the answer is 59"

Use the formal written method for addition.



			
Column method – regrouping	Using base 10/Deinnes blocks and place value counters, make both numbers on a place value grid. Add up the ones/units and exchange ten ones/units for one ten. 	Represent base 10/Deinnes blocks in a place value chart and exchange appropriately.  Draw a pictorial representation of the columns and place value counters. 	Start by partitioning the numbers before adding. $ \begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array} $ Use the formal written method for addition. Please note that the below shows the use of a 'Just in case space' above the total for children to record any 'carried' digits. However, children should also be exposed to carried digits being recorded below the line too.

**Column method
– regrouping
(cont.)**



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

As children move on to decimals, money and decimal place value counters should be used.

$$34 + 18 = ?$$

$$\begin{array}{r} 34 \\ + 18 \\ \hline 1 \\ \hline 52 \end{array}$$

so $34 + 18 = 52$

$$\begin{array}{r} 587 \\ + 375 \\ \hline 11 \\ \hline 962 \end{array}$$

Repeat the above with decimal numbers, with both the same and different number of decimal places

$$\begin{array}{r} 123.9 \\ + 7.25 \\ \hline 11 \\ \hline 131.15 \end{array}$$

Mathematical variation and the impact upon this policy.

Variation theory is a way of analysing and planning teaching and learning activities. The approach focuses on what changes, what stays the same and the effect this might have.

There should be different emphases for different critical aspects when structuring variation in mathematics lessons, and learners should see difference before sameness, including counter or non-examples.

The models and images above, alongside appropriate mathematical variation, will help teachers to structure tasks to direct pupil attention most effectively. The patterns of variation include:

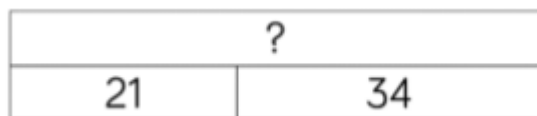
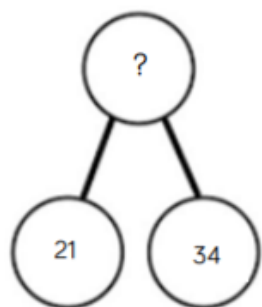
Contrast – To experience something we must experience something else to compare it with.

Generalisation – Experiencing and recognising varying appearances of the same thing (for example the different pictorial representations above)

Separation – Seeing one aspect as distinct from other aspects.

Fusion – Experiencing several critical aspects simultaneously (often called co-variation).

Conceptual variation (different ways to ask children to solve $21+34$)



Word problems: In Year 3, there are 21 children in Year 4 there are 34 children. How many children are there in total?

$$21 + 34 = 55. \text{ Prove it.}$$

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\boxed{} = 21 + 34$$

Calculate the sum of twenty-one and thirty-four.



Fill in the missing digits.

10s	1s
	?
?	5